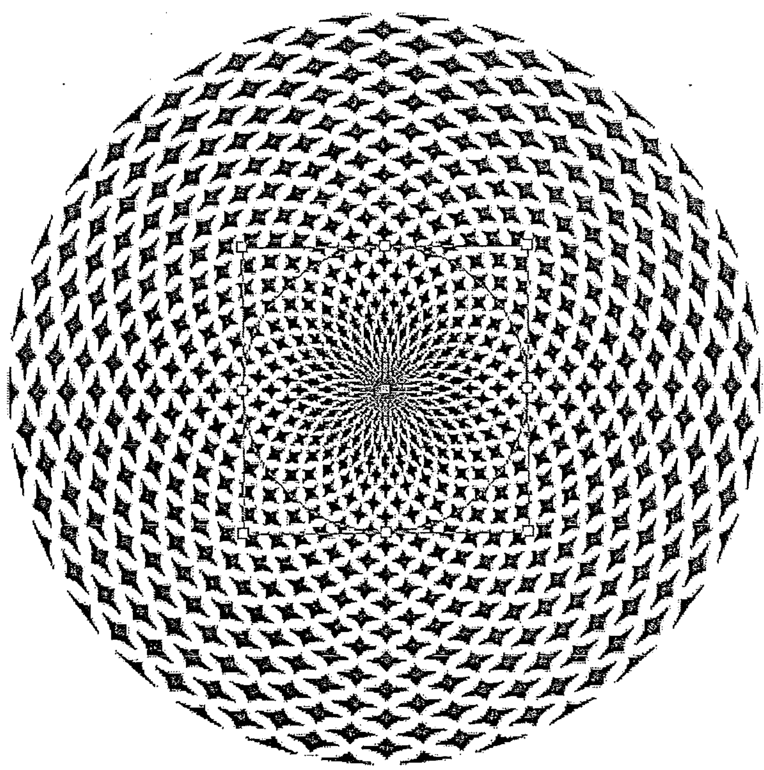
KEY

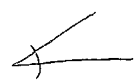
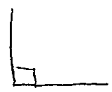
Math 9
Chapter 9 Circle Geometry

Notes

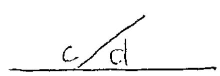
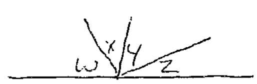
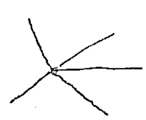
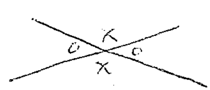


Math 9 Geometry Introduction Name:

There are four different kinds of angles. Can you name them and draw them?

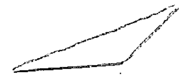
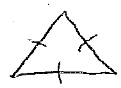
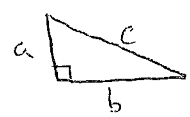
Name	Sketch	Explanation
Acute		less than 90°
Obtuse		greater than 90°
Right		90°
Straight		180°

There are five important rules that relate lines to each other
Name them, then sketch and explain them

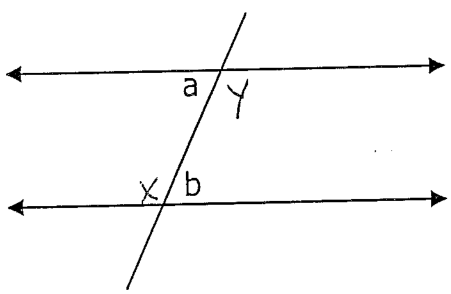
Name	Sketch	Explanation
Complementary		∡'s a & b add up to 90°
Supplementary		∡'s c and d add up to 180°
Angles on a line		angles w, x, y, z on a line add up to 180°
Angles @ a point		angles @ a point add up to 360°
Vertically opposite		vertically opposite ∡'s are equal

There are four different kinds of triangles. Can you identify them?

Recall → ∡ sum of a triangle is 180°

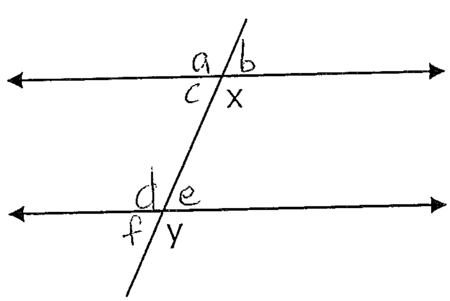
Name	Sketch	Explanation
Scalene		- no equal sides - no ∡'s equal
Isosceles		- at least 2 sides equal - angles opposite the equal sides are equal
Equilateral		- 3 sides equal - 3 ∡'s are equal (60°)
Right		- 1 right ∡ (90°) - hypotenuse is opposite right ∡ - $a^2+b^2=c^2$

Parallel Lines create some useful geometry rules, when they are intersected by a line called a TRANSVERSAL



Angles a and b are a pair of ALTERNATE INTERIOR ANGLES. Can you find another pair? Look for the 'Z' shape $\angle a = \angle b$

$\angle x = \angle y$



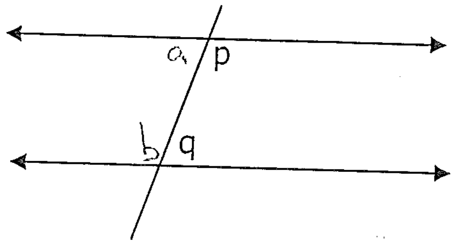
Angles x and y are a pair of equal CORRESPONDING ANGLES. How many other pairs are there? $\angle x = \angle y$

Look for the 'F' shape

$\angle a = \angle d$

$\angle b = \angle e$

$\angle c = \angle f$



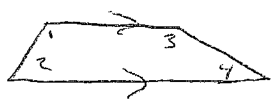
Angles p and q are called CO-INTERIOR ANGLES.
They add to 180° ($\angle p + \angle q = 180°$)
Is there another pair? Look for the 'C' shape

$\measuredangle a + \measuredangle b = 180°$

What can you remember about Quadrilaterals? How many types are there?

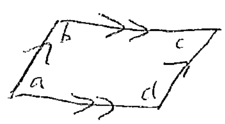
→ $\measuredangle$ sum of a quadrilateral is 360°

Trapezoid



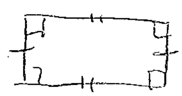
- 1 pair of parallel lines (|| → sign for parallel)
- $\measuredangle 1 + \measuredangle 2 = 180°$, $\measuredangle 3 + \measuredangle 4 = 180°$

Parallelogram



- opposite sides are equal and parallel
- opposite $\measuredangle$'s are equal ($\measuredangle a = \measuredangle c$, $\measuredangle b = \measuredangle d$)
- consecutive $\measuredangle$'s equal 180°

Rectangle

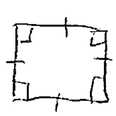


- opposite sides are equal & parallel
- each $\measuredangle$ is 90°

Rhombus

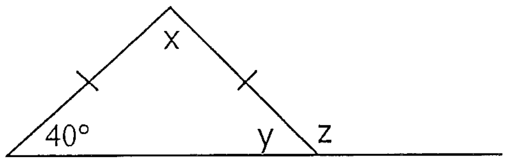
- parallelogram w/ 4 equal sides

Square



- rhombus with 4 right $\measuredangle$'s

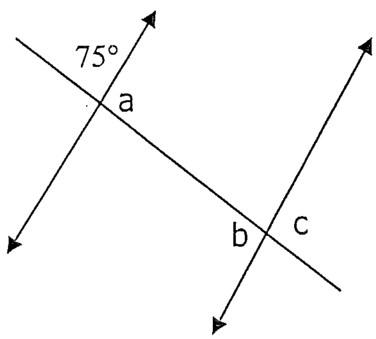
Practice. Find the missing angles and give the best reason.



y = 40° ∡'s opposite = sides are =

x = 100° ∡'s in a Δ add up to 180°

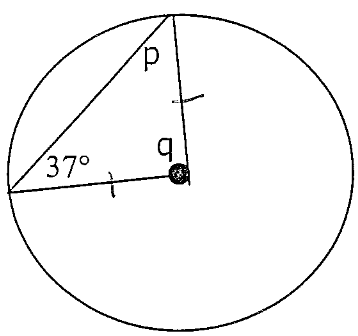
z = 140° supplementary / ∡'s on a line add up to 180°



a = 105° supplementary / ∡'s on a line add up to 180°

b = 105° alt-interior ∡'s are =

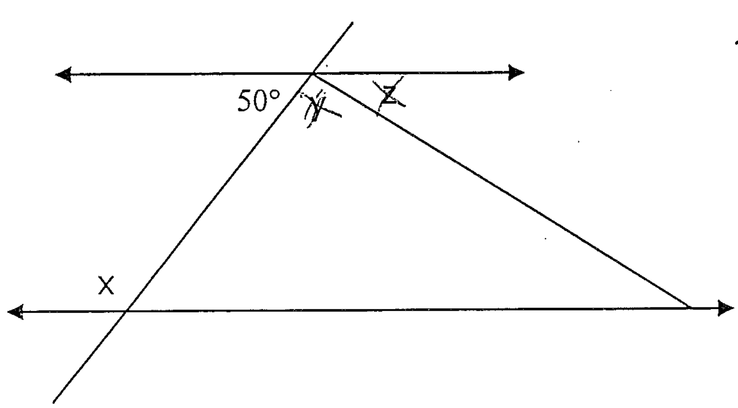
c = 105° vertically opposite ∡'s are =



lines from centre to circle are = (radius)

∡p = 37° ∡'s opposite = sides are =

∡q = 106° ∡'s in a Δ add up to 180°



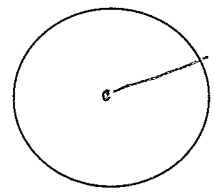
∡x = 130° Co-interior ∡'s add up to 180°

Math 9 The Chord Property Name:

Definitions:

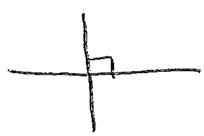
Circle a set of points that are the same distance from the centre

Radius



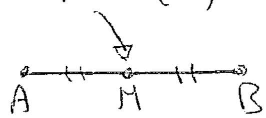
distance from centre to circle

Perpendicular:

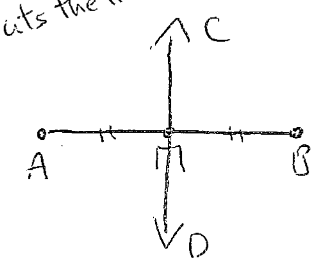


intersecting lines @ right angles

Midpoint: (M)

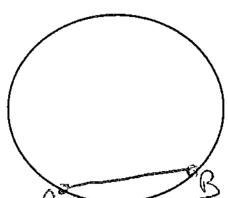


Bisect: - cuts the line in half



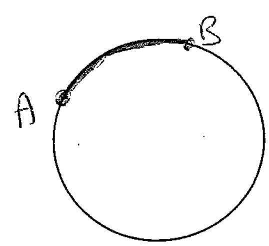
CD bisects AB

Chord:



a line joining two ~~segment whose end~~ points on a circle

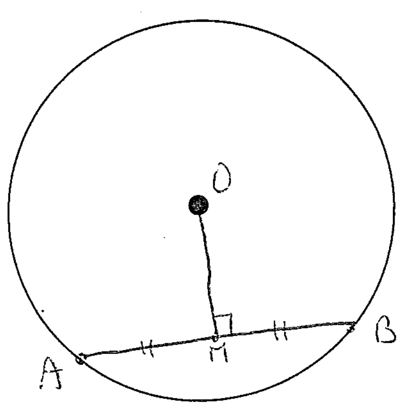
Arc:



part of a circle

$\overset{\frown}{AB}$

Draw a chord AB on this circle. Connect the midpoint M of the chord to the center O. Measure $\angle OMA$ and $\angle OMB$. Record your result

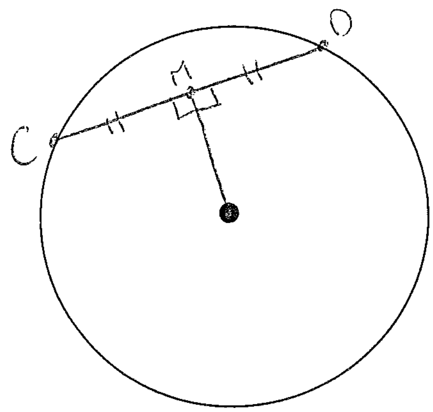


$\angle OMA = 90°$ $\angle OMB = 90°$

- a line from the centre bisects the chord AB creating perpendicular lines

C

Draw a different length chord CD, in a different location. Draw a segment from the centre to the chord at right angles to the chord. Label the point where they intersect M. Measure CM and DM. What did you find?

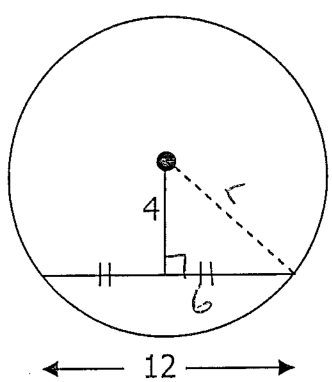


CM = DM

∴ OM is a perpendicular '⊥' bisector

Practice:

Find the length of the radius.



- use pythagoras $a^2+b^2=c^2$

$r^2=4^2+6^2$

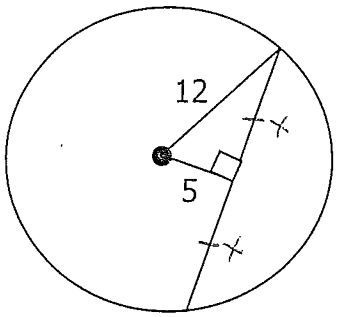
$r^2=16+36$

$r^2=52$

$\sqrt{r^2}=\sqrt{52}$

$r \approx 7.21$

Find the length of the chord



$x^2 + 5^2 = 12^2$

$x^2 + 25 = 144$

$-25 \quad -25$

$x^2 = 119$

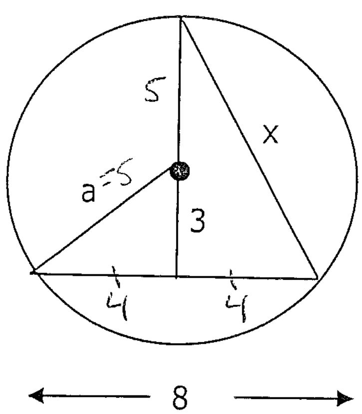
$\sqrt{x^2} = \sqrt{119}$

$x \approx 10.9$

$\therefore$ chord $= 2 \times 10.9$

$= 21.8$

Find the value of x



step 1

$a^2 = 3^2 + 4^2$

$a^2 = 9 + 16$

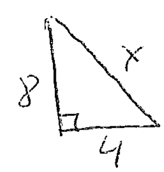
$a^2 = 25$

$\sqrt{a^2} = \sqrt{25}$

$a = 5$

radius

step 2



$x^2 = 4^2 + 8^2$

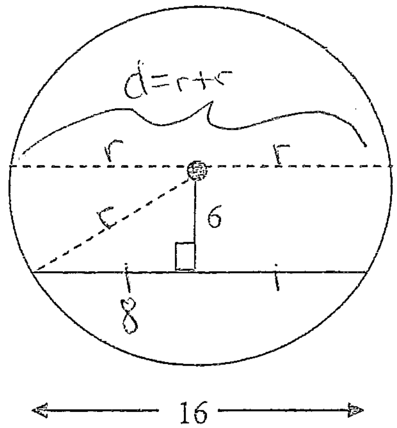
$x^2 = 16 + 64$

$x^2 = 80$

$\sqrt{x^2} = \sqrt{80}$

$x \approx 8.9$

1)

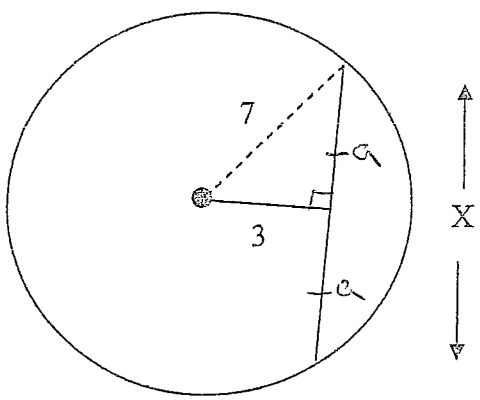


Find the length of the diameter
HINT: Find the radius first

$r^2 = 6^2 + 8^2$
$r^2 = 36 + 64$
$r^2 = 100$
$r = 10$

$d = r + r$
$= 10 + 10$
$= 20$

2)

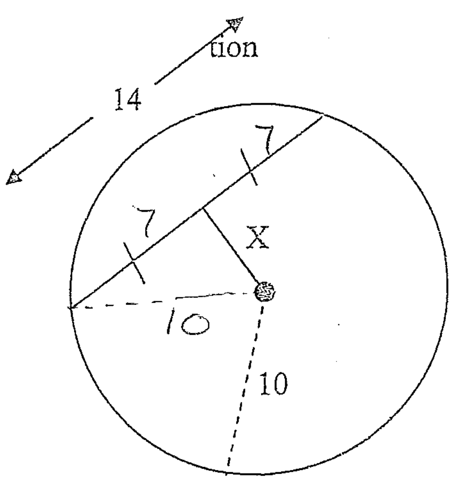


Find the length of the chord
HINT: Find half of the chord first

$7^2 = 3^2 + a^2$
$49 = 9 + a^2$
$-9 \quad -9$
$40 = a^2$
$\sqrt{40} = \sqrt{a^2}$
$6.3 = a$

$x = a + a$
$= 6.3 + 6.3$
$= 12.6$

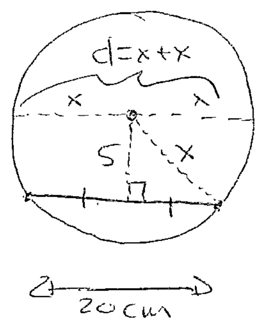
3)



Find the length of X
HINT: Create a right triangle

$10^2 = 7^2 + x^2$
$100 = 49 + x^2$
$-49 \quad -49$
$\sqrt{51} = \sqrt{x^2}$
$7.1 = x$

4) A cylindrical pipe has a circular cross section. The water at the bottom of the pipe is 20 cm wide. The water is 5 cm from the center of the pipe. What is the diameter of the pipe?



$x^2 = 5^2 + 10^2$

$x^2 = 25 + 100$

$\sqrt{x^2} = \sqrt{125}$

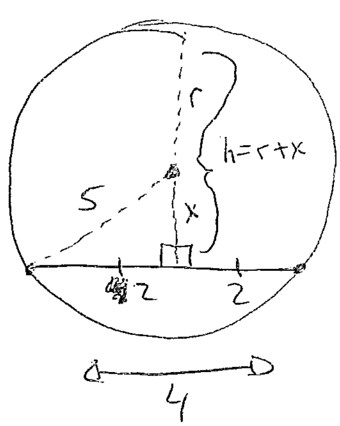
$x = 11.2\text{cm}$

$d = x + x$

$= 11.2 + 11.2$

$= 22.4\text{cm}$

5)A road underpass is shaped as a circle. The underpass has a radius of 5 feet, while the pathway inside is 4 feet wide. What is the maximum height of the underpass?



$5^2 = x^2 + 2^2$

$25 = x^2 + 4$

$-4 \qquad -4$

$\sqrt{21} = \sqrt{x^2}$

$4.6 = x$

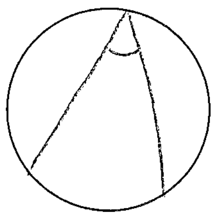
height $= x +$ radius

$= 4.6 + 5$

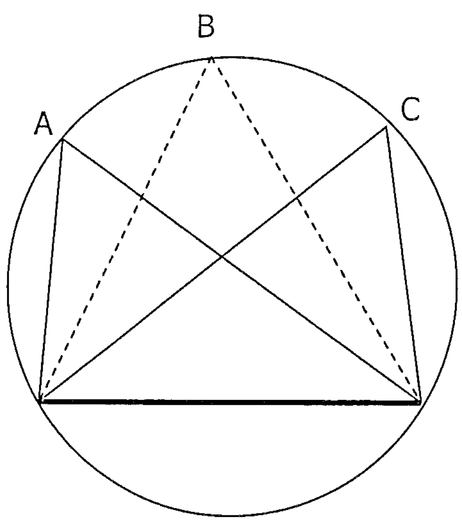
$= 9.6$ ft.

Math 9 Inscribed Angles Name:

Definitions: Inscribed angle: is an angle whose "vertex" is on the circle



Three hockey players face the net in a circular rink. Which player has the best angle at which to shoot?



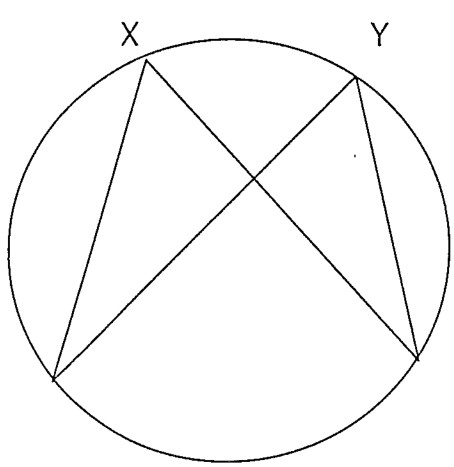
$\angle A = 60°$

$\angle B = 60°$

$\angle C = 60°$

$\therefore$ all 3 inscribed $\angle$'s are =

Measure the inscribed angles X and Y. What can you conclude?

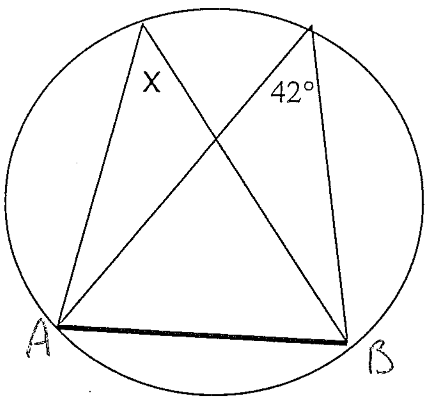


$\angle X = 59°$

$\angle Y = 59°$

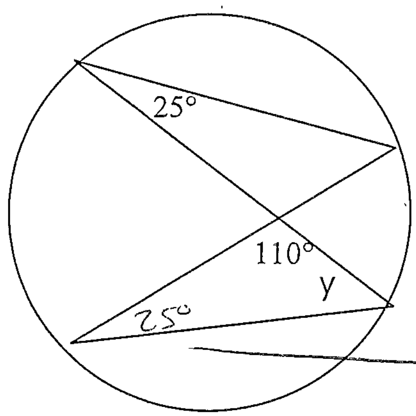
$\therefore$ inscribed $\angle$'s covered by the same chord or arc are equal

Practice:



$\measuredangle x = 42°$

↳ both are inscribed angles 'sharing' the same chord AB

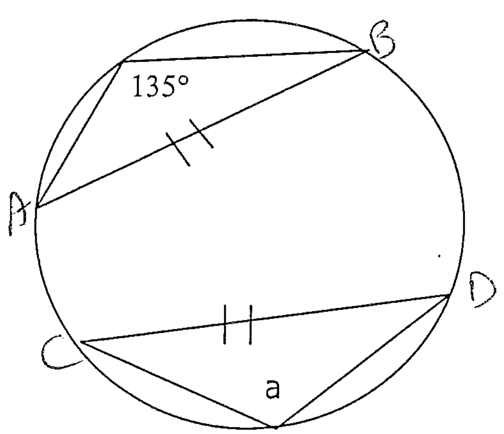


$\therefore \measuredangle y = 45°$

$(180° - 110° - 25° = 45°$

$\measuredangle$'s in $\triangle$ add to $180°)$

shares an arc with 25°

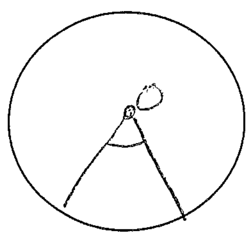


$\measuredangle a = 135°$

* both inscribed angles are 'covered by' equal chords $\overline{AB}$ and $\overline{CD}$

Math 9 Central Angle Notes Name: ______________________

Definition: Central Angle → an angle whose vertex is at the centre



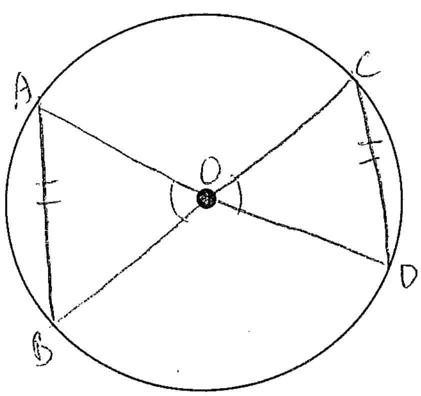
Draw two equal chords, one on either side of the circle. Label them AB and CD. Label center as O. Create central angles $\angle AOB$ and $\angle COD$ by joining A and B to O. and C and D to O.

Measure $\angle AOB$ and $\angle COD$. What is your conclusion?

$\angle AOB = 60°$ $\angle COD = 60°$

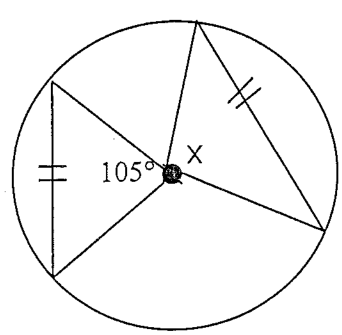
Conclusion: ∴ both ∡AOB & ∡COD are equal

↳ central ∡'s with equal chords will have = ∡'s



Practice:

1. Find the measure of x



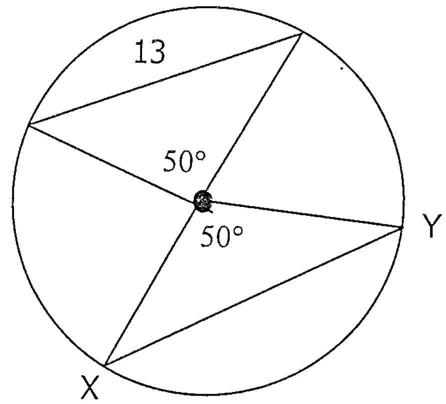
∡x = 105°

both central angles are 'covered by' = chords

Find the length of chord XY

$XY = 13$

↳ central ∡s are =, thus chords must be = length



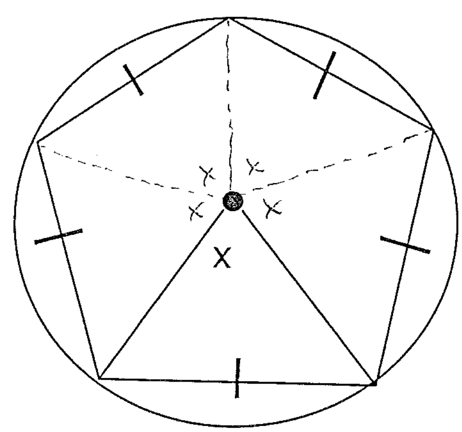
Find the measure of X

angles around a pt add to 360°

$X + X + X + X + X = 360°$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{360°}{5}$

$x = 72°$



Find the measure of Y

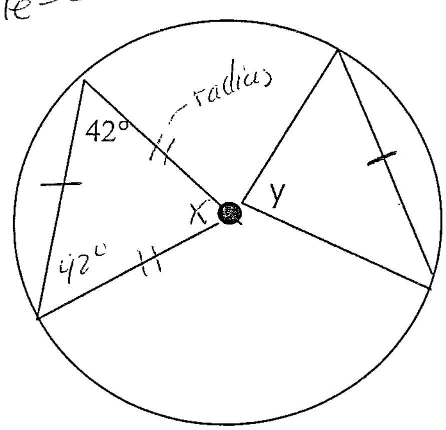
Δ's are = opposite = sides

$\measuredangle x = 180° - 42° - 42°$

$= 96°$

$\therefore \measuredangle y$ is also $96°$

↳ equal chords

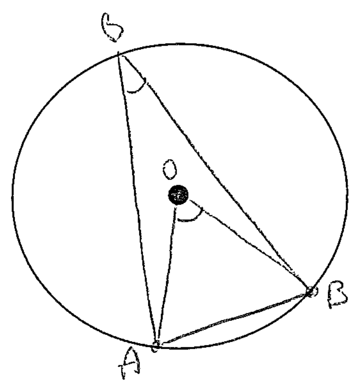


Math 9 Central/Inscribed Property Name:

This lesson is designed to discover the relationship between and central and an inscribed angle

Draw chord AC. Draw inscribed angle ABC. Draw central angle AOC.
Measure $\angle ABC$ and $\angle AOC$. Record your results.

Both ∡'s "share" the same chord.

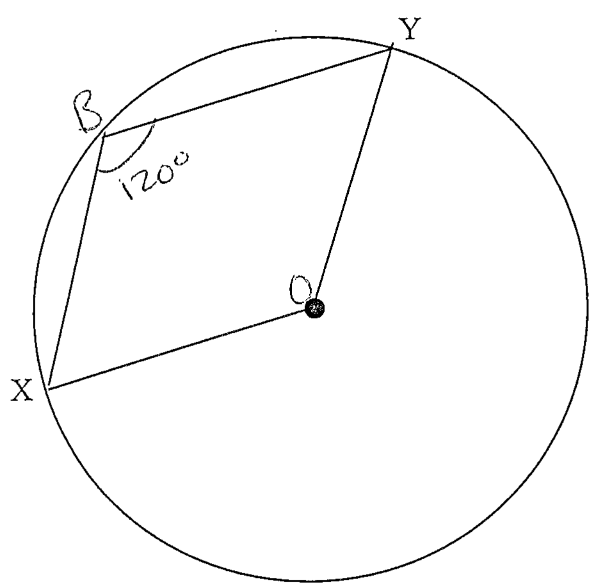


$\angle ABC =$ 37° (inscribed ∡) $\angle AOC =$ 74° (central ∡)

Conclusion:

The central ∡, when ~~connected~~ ~~when using a~~ sharing the same chord, was double the size of the inscribed ∡

Repeat this experiment using an arc XY.



∡XBY (inscribed) = 120°

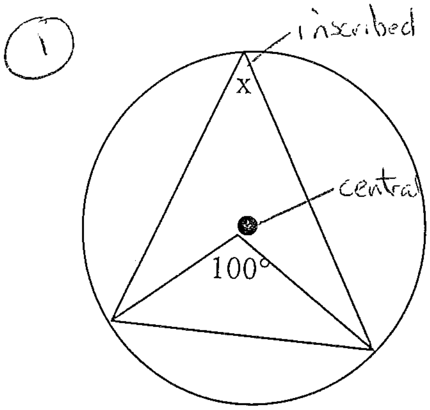
∡XOY (central) = 2 × 120° = 240°

∴ the same conclusion as before works on equal/same arcs

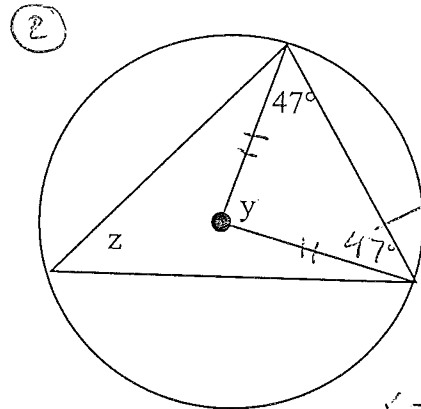
Do you have the same result? Yes

central ∡ is 2× inscribed ∡ on equal/same chords/arcs.

Practice:



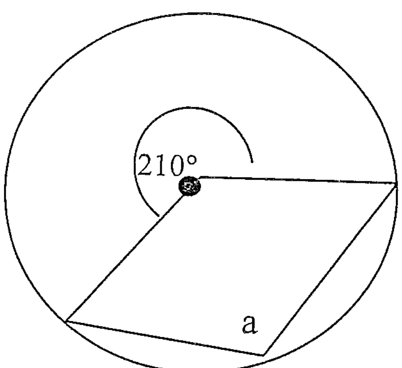
$\measuredangle x = 100° \div 2$
$= 50°$ (inscribed ½ of central ∠)



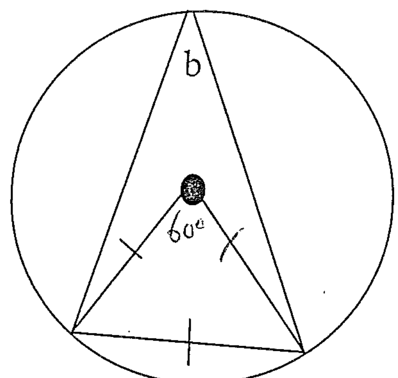
∠'s opp = sides are =

$\measuredangle y = 180 - 47 - 47$
$= 86°$ (∠'s in a △)

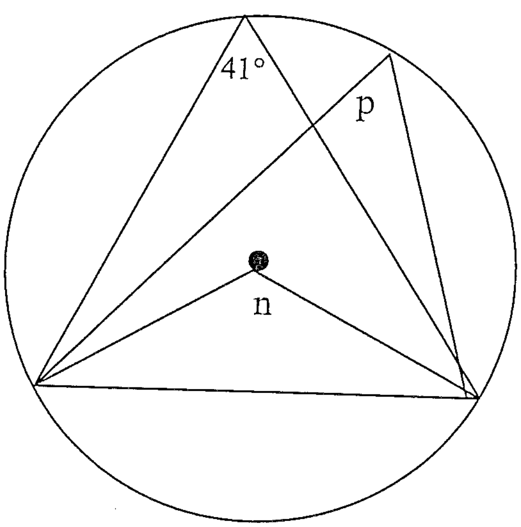
$\measuredangle z = 86 \div 2$
$= 43°$



$\measuredangle a = 210° \div 2$
$= 105°$



→ equilateral △, all ∠'s 60°
→ $\measuredangle b = 60° \div 2 = 30°$

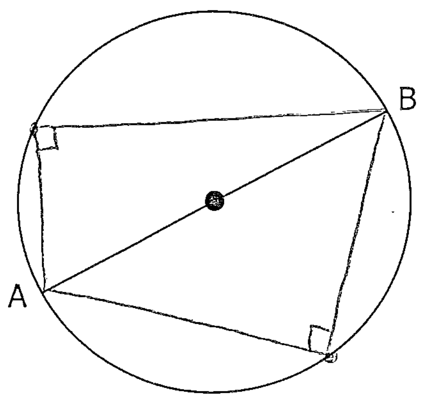


$\measuredangle p = 41°$ (inscribed ∠ on same chord)
$\measuredangle n = 2 \times 41°$
$= 82°$ (central ∠ double inscribed ∠)

Math 9 Semi-Circle Property Name:

In this lesson we will explore the properties of an inscribed angle covered by the diameter

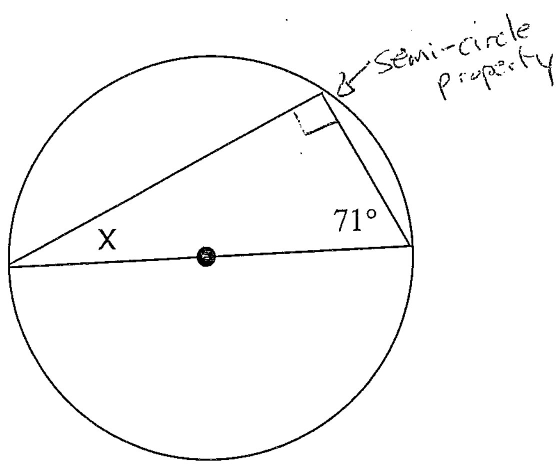
Draw inscribed angle APB. Measure and record $\angle APB$ $\angle APB =$ 90°

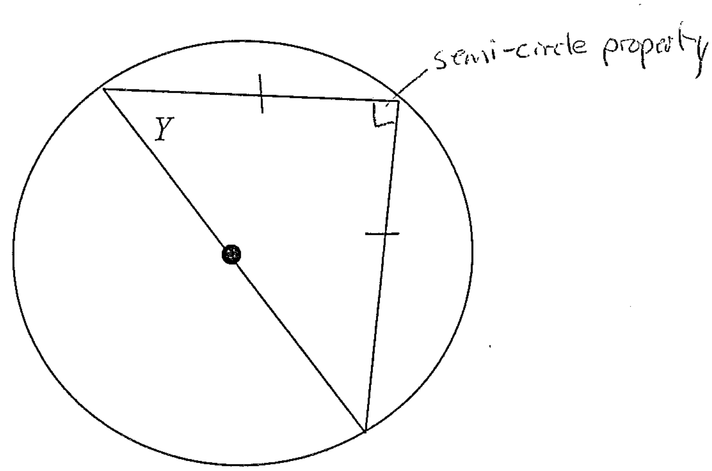


Draw inscribed angle AQB. Measure and record $\angle AQB$ $\angle AQB =$ 90°

Conclusion: an ~~is~~ inscribed ∡ in a semi-circle is 90°

Practice:





△ sum = 180°

$\measuredangle x = 180° - 90° - 71°$

$= 19°$

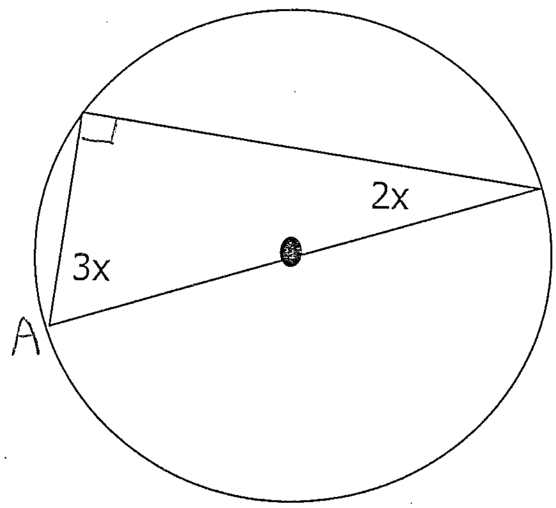
$y + y + 90° = 180°$

$2y + 90° = 180°$

$\frac{2y}{2} = \frac{90°}{2}$

$y = 45°$

16



Δ sum $= 180°$

$3x + 2x + 90° = 180°$

$5x + 90° = 180°$

$\frac{5x}{5} = \frac{90°}{5}$

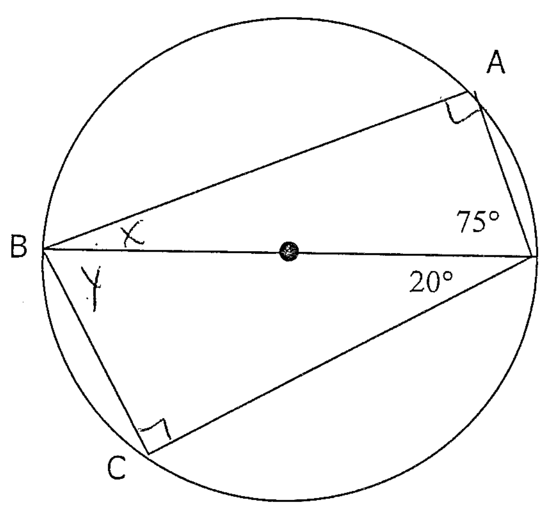
$x = 18°$

X = 18°

$\measuredangle A = 3x$

$= 3(18°)$

$= 54°$



$\measuredangle x = 180° - 90° - 75°$

$= 15°$

$\measuredangle y = 180° - 90° - 20°$

$= 70°$

$\angle ABC = \measuredangle x + \measuredangle y$

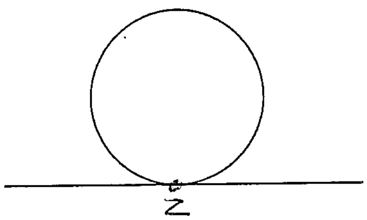
$= 15° + 70°$

$= 85°$

Ma 9 Tangent Radius Property Name:

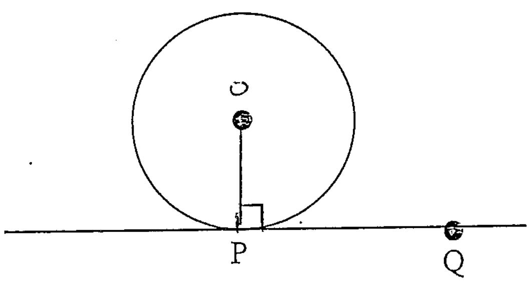
Goal is to explore the relationship between a tangent and its radius

Definition: A tangent is a line that intersects a circle at exactly one point



e.g. point Z is called the "point of tangency".

Draw a segment joining the center with the point of tangency. Measure angle OPQ

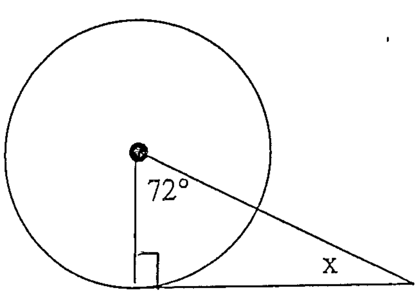


$\angle OPQ =$ $\underline{90^\circ}$

Conclusion:

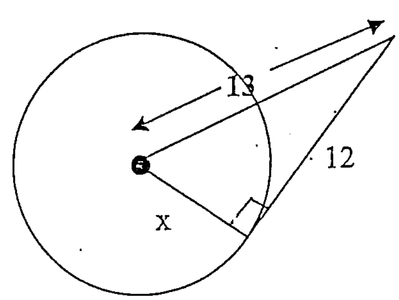
A tangent to a circle is perpendicular. (90° angle) to the radius at the point of tangency.

Practice:



$x = 180 - 72 - 90$

$x = 18^\circ$

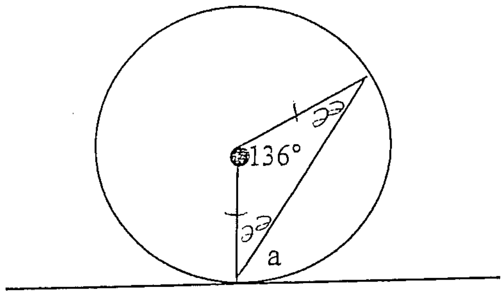


$a^2 + b^2 = c^2$

$x^2 + 12^2 = 13^2$

$x^2 = 13^2 - 12^2$

$x^2 = 169 - 144$



180 - 136 = 44 (∠'s in a △)

44 ÷ 2 = 22 (isosceles △)

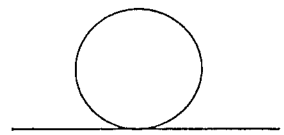
∠a = 90 - 22 (tangent radius property)

= 68°

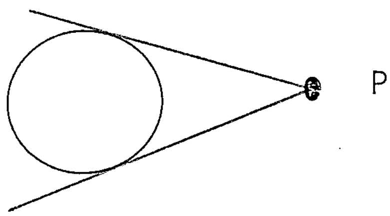
Math 9 Tangents from an External Point Name:

The goal is to explore the relationship between tangents drawn to a circle

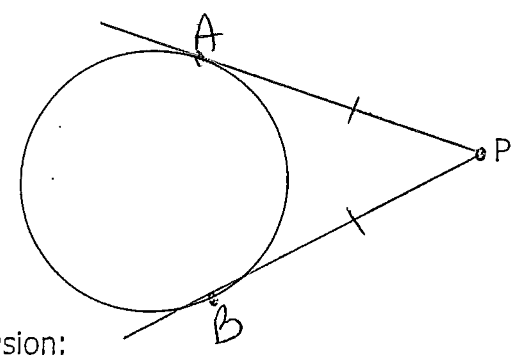
Remember---A tangent is a line that intersects a circle once



How many tangents can be drawn to a circle from point P? 2.



Draw tangents from point P to the circle. Label the points of tangency as A and B. Measure the lengths of PA and PB. Record the result. What is your conclusion?



PA = 4cm

PB = 4cm

Conclusion:

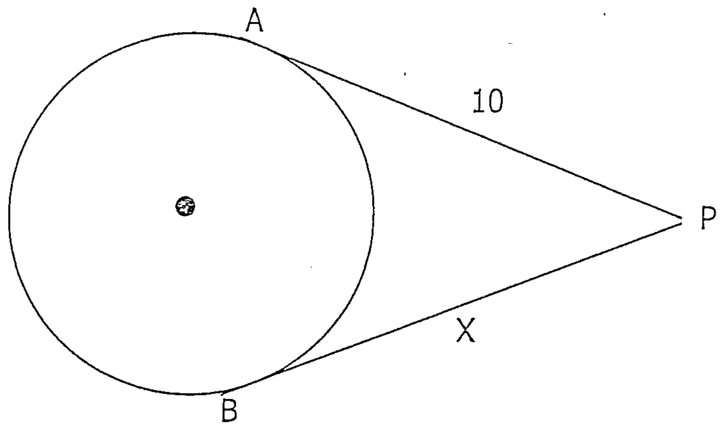
PA = PB

Tangent segments drawn from an external point to a circle are equal

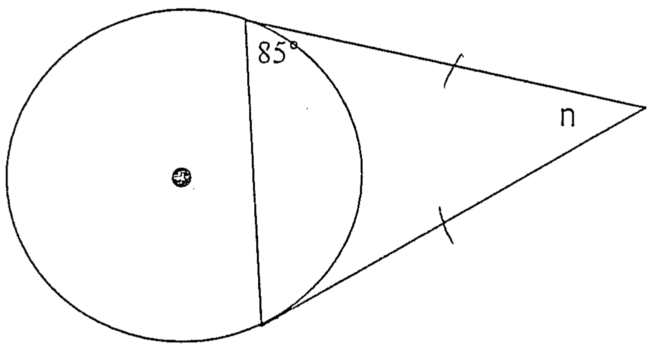
Equal Tangents Property:

Tangent properties and the Pythagorean theorem can be used to solve circle problems.

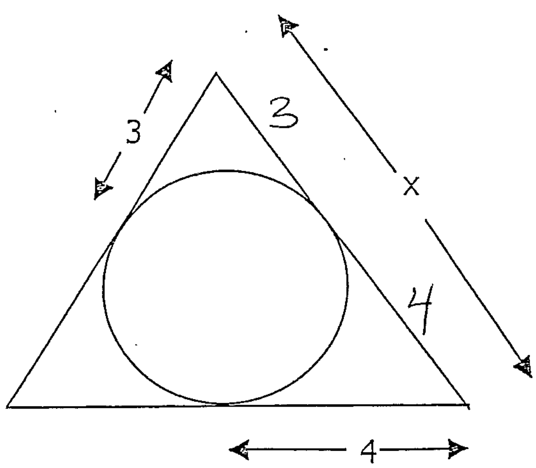
Practice:



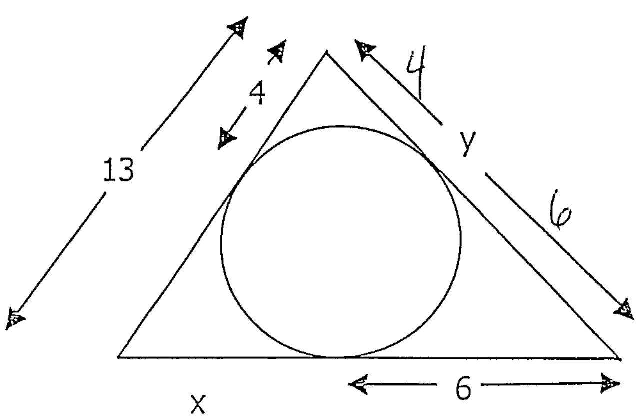
$x = 10$



$n = 180 - 85 - 85$
$= 10°$



$x = 3 + 4$
$= 7$



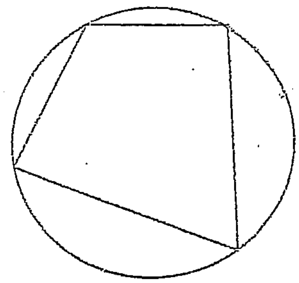
Perimeter = $13 + 4 + 6 + 6 + 9$
$= 38$

$13 - 4 = 9$

Math 9 Cyclic Quadrilateral Name:

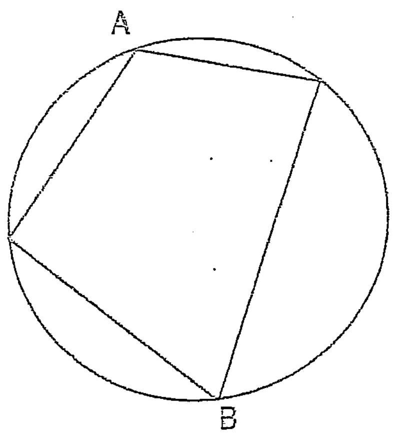
The goal is to explore the angles in a Cyclic Quadrilateral

Definition: A Cyclic Quadrilateral is: - a quadrilateral (4 sides) with all 4 vertices on a circle.



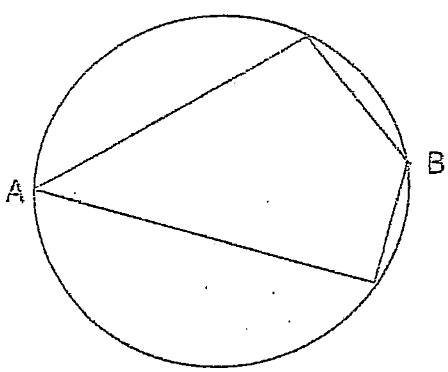
- a quadrilateral drawn inside a circle so that its corners lie on the circumference of the circle.

Measure angles A and B. Record your results



$\angle A = 110^\circ$

$\angle B = 70^\circ$



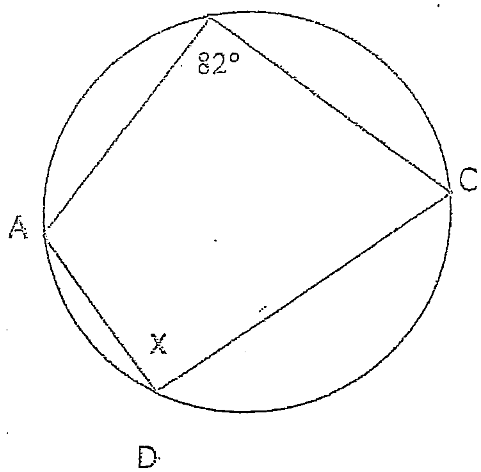
$\angle A = \cancel{40}\ 50^\circ$

$\angle B = 130^\circ$

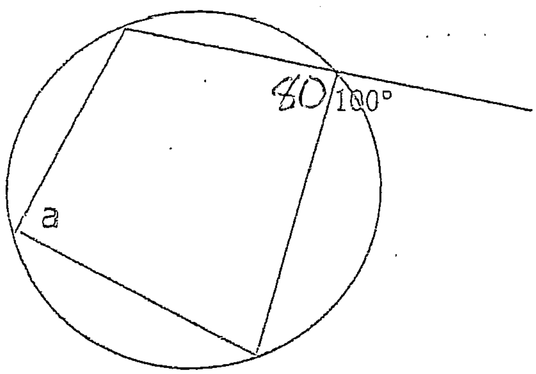
Cyclic Quadrilateral Property:

Opposite angles of a cyclic quadrilateral add to 180°

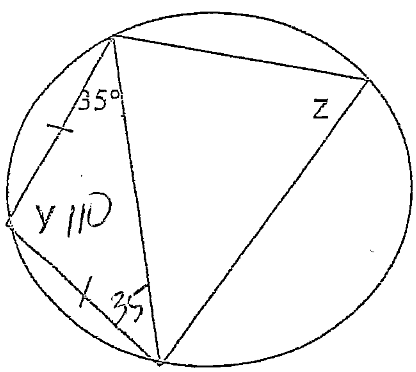
Practice:



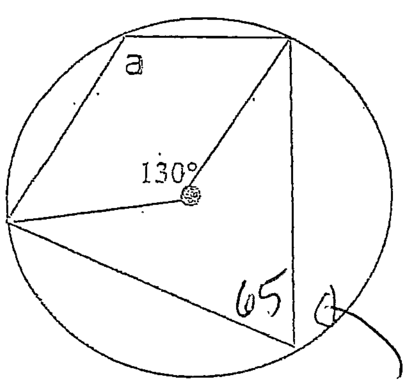
$\angle x = 180 - 82$
$= 98°$



$\angle a = 100°$



$\angle y = 110°$ ($\angle$'s in a $\triangle$)
$\angle z = 70°$ (cyclic quadrilateral)



$\angle a = 180 - 65$
$= 115°$

central / inscribed