

Key

Everyday, we encounter statements such as:

20% chance of rain

Chance of winning a prize is 1 in 50

These are examples of probability.

Probability is the LIKELIHOOD that an event will occur. Any outcome or set of outcomes, of an experiment is called an EVENT

There are two kinds of probabilities:

Experimental Probability: determined by actually performing the experiment such as flipping a coin, drawing a playing card, or rolling a die

Theoretical Probability: (The probability which would occur in a perfect world) and is determined by using a formula.

$$P(A) = \frac{\# \text{ of favorable outcomes}}{\# \text{ of total outcomes}}$$

Outcomes you hope for

or are interested in.

EXPERIMENTAL PROBABILITY

What is the THEORETICAL PROBABILITY of flipping a coin?

$\frac{1}{2}$, 0.5, 50%, 50 – 50

Flip a coin 10 times, each time recording what you get and fill in the table with the results.

Flip	1	2	3	4	5	6	7	8	9	10
Result										

Calculate your EXPERIMENTAL PROBABILITY of flipping 'tails'.

Number of Tails
Total # of Flips

$$= \frac{\quad}{10}$$

Fill in a table for the entire class

Total ^{Tails} Trials	Total Flips	$\frac{53}{120}$
53	120	

= 0.44

What is the experimental probability of flipping tails for the class?

$\frac{53}{120}$ or 0.44 or 44%

How do the three probabilities compare?

Theoretical: (0.5), yours (experimental with 10 trials), and

The class (experimental with $\frac{0.44}{120}$ trials)

An EXPERIMENTAL PROBABILITY will get close to the THEORETICAL PROBABILITY after many trials

THEORETICAL PROBABILITY

$P(A) = \frac{\# \text{ of favorable outcomes}}{\# \text{ of total outcomes}}$

What you want to happen

EXPERIMENTAL PROBABILITY

$P(A) = \frac{\# \text{ of observed favorable trials}}{\# \text{ of trials}}$

Real

VOCABULARY

Outcome	result of an event
Event	1 or more outcomes (for example- flipping a coin)
Probability of an outcome	The chance that it will occur
Probability given as a	Ratio $\frac{1}{4}$; Decimal 0.25; Per Cent 25 %
0% = no chance; 100% = a certainty; 50% = Equal chance	

Black
CLUBS
SPADES

Red
HEARTS
DIAMONDS

A 2 3 4 5 6 7 8 9 10 J & K
13 cards in each suit
52 cards total

EXAMPLES

A card is drawn from a deck of 52 cards. Find the THEORETICAL PROBABILITY that the card is:

a) a club

$$P(\text{a club}) = \frac{13}{52} = \frac{1}{4} = 0.25 = 25\%$$

b) an ace

$$P(\text{Ace}) = \frac{4}{52} = \frac{1}{13} = 0.08 = 8\%$$

c) a red card

$$P(\text{red}) = \frac{26}{52} = \frac{1}{2} = 0.5 \text{ or } 50\%$$

d) a 7 of clubs

$$P(7 \text{ of clubs}) = \frac{1}{52} = 0.02 = 2\%$$

What is the THEORETICAL PROBABILITY of rolling a single die having it come up an even number?

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$$

1
2
3
4
5
6

A bag contains 3 blue marbles, 2 yellow marbles, and 4 green marbles. What is the THEORETICAL PROBABILITY of choosing a blue marble?

$$\begin{aligned} P(\text{blue}) &= \frac{3}{9} = \frac{1}{3} \\ &= 0.33 \\ &= 33\% \end{aligned}$$

↓↓ ↓

One letter is chosen randomly from the word MATHEMATICS. What is the probability of choosing? ~~11~~ 11 letters

a) An 'm'

$$\begin{aligned} P(M) &= \frac{2}{11} \\ &= 0.18 \\ &= 18\% \end{aligned}$$

b) An 'e'

$$\begin{aligned} P(E) &= \frac{1}{11} \\ &= 0.09 \\ &= 9\% \end{aligned}$$

c) A vowel

$$\begin{aligned} P(\text{vowel}) &= \frac{4}{11} \\ &= 0.36 \\ &= 36\% \end{aligned}$$

d) A 't' or an 'h'

$$\begin{aligned} P(T \text{ or } H) &= \frac{3}{11} \\ &= 0.27 \\ &= 27\% \end{aligned}$$

Pm 9

7.1 ws2 Single Event Name: *Key*

Calculate the theoretical probability of the following

1. Rolling an odd number on a single 6-sided die $P(\text{odd}) = \frac{1}{2}$
2. Choosing 1 blue marble from a jar containing 5 red, 4 blue and 6 white. $P(\text{blue}) = \frac{4}{15}$
3. Choosing a Jack from a deck of 52 well shuffled cards $P(\text{Jack}) = \frac{4}{52} = \frac{1}{13}$
4. Choosing the letter 'a' from the word Saturday $P(a) = \frac{2}{8} = \frac{1}{4}$
5. Choosing a red Ace from a deck of 52 well shuffled cards $P(\text{red Ace}) = \frac{2}{52} = \frac{1}{26}$
6. From a jar containing 6 red balls, 3 green ones, 4 orange, and 7 yellow ones.
What is the probability of choosing 1 orange ball? $P(\text{orange}) = \frac{4}{\frac{52}{20}} = \frac{1}{5}$
7. What is the probability that a knight is chosen from the 16 white chess pieces
 $P(\text{knight}) = \frac{2}{16} = \frac{1}{8}$
8. Choosing an even numbered card from a deck of 52 well shuffled cards
 $P(\text{even}) = \frac{20}{52} = \frac{5}{13}$
9. Choosing a day that ends in a 'y' from the days of the week $P(y) = \frac{7}{7} = 1$ 100%
10. Rolling an 8 – sided die and having it turn up a prime number (don't include 1 as a prime)
$$\begin{aligned} P(\text{prime}) &= \frac{4}{8} \\ &= \frac{1}{2} \\ &= 0.5 \\ &= 50\% \end{aligned}$$

9.4 The Probability Formula

MATHPOWER™ Nine, pp. 348-349

When all the outcomes are known and equally likely, the probability of a single outcome is given by the probability formula.

$$\text{Probability of an outcome} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

1. Choose one card without looking.

A B C D E F

Find the probability of choosing:

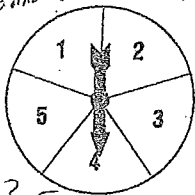
- a) a vowel $\frac{1}{3}$ b) B, C, or D $\frac{1}{2}$
 c) a consonant $\frac{2}{3}$ d) A, B, C, D, E, or F 1 or 100%

2. In question 1, which letters have the following probabilities?

- a) 0 none b) $\frac{1}{6}$ A, B, C, D, E, F

3. What is the probability of spinning each of the following numbers with this spinner? Express each answer as a percent.

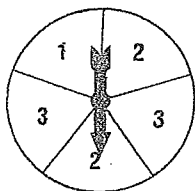
- a) an even number $\frac{2}{5}$ *or # that cannot be divided by a # other than 1 & itself*
 b) a prime number $\frac{3}{5}$
 c) 5 $\frac{1}{5}$ *2, 3, 5*



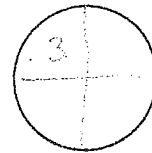
- d) 1, 2, or 3 $\frac{3}{5}$

4. What is the probability of spinning each of the following numbers with this spinner? Express each answer as a decimal.

- a) an even number $\frac{2}{5}$
 b) a prime number $\frac{4}{5}$
 c) 5 \emptyset
 d) 1, 2, or 3 1



5. If the probability of spinning a 3 on a circular spinner is 0.25, draw the simplest way the spinner can be divided on the circle.



6. One ball is drawn from a bag containing 1 yellow, 2 orange, 3 pink, and 2 white marbles. State the probability of the following colours. Write your answers in decimal form. *total = 8*

- a) P(white) $\frac{2}{8} = \frac{1}{4}$ b) P(pink) $\frac{3}{8}$
 c) P(yellow) $\frac{1}{8}$ d) P(orange) $\frac{2}{8} = \frac{1}{4}$

7. In the game of SCRABBLE™, there are 100 tiles distributed as follows.

A-9 B-2 C-2 D-4 E-12 F-2
 G-3 H-2 I-9 J-1 K-1 L-4
 M-2 N-6 O-8 P-2 Q-1 R-6
 S-4 T-6 U-4 V-2 W-2 X-1
 Y-2 Z-1 Blank-2

If all the tiles are placed in a bag and one tile is drawn from the bag, find the probability of drawing the following. Write each answer as a percent.

- a) P(G) $\frac{3}{100}$ b) P(A) $\frac{9}{100}$
 c) P(Blank) $\frac{2}{100}$ d) P(O) $\frac{8}{100}$

8. In question 7, which letters have the following probabilities?

- a) 6% N, R, T b) 1% J, K, Q, Z, X

9. Which letter in question 7 has the highest probability of being chosen? What is the probability? Write your answer as a percent.

E, 12%

key

Pm 9

7.1 Recognizing Probability Situations Name:

In groups of four, determine first the theoretical, then the experimental probability of each situation. Provide a sufficient number of trials for the experimental probability portion of the exercise.

Show ALL work and write the probability statements in all forms:

a) Ratios

b) Decimals

c) Per Cents

A. What is the probability of obtaining a 3 with the roll of a single die?

Theoretical

Exp

a) $1/6$

b) $0.1\bar{6}$

c) 16.7%

B. What is the probability of obtaining a sum of 8 with the roll of 2 dice?

Theoretical

Exp

~~2/6~~ $2 \& 6 = 8, 3 \& 5 = 8, 4 \& 4 = 8$
 $6 \& 2 \quad 5 \& 3 \quad 4 \& 4$

a) ~~$6/36 = 1/6$~~ $5/36$

b) $0.13\bar{8}$

c) 13.8%

C. What is the probability of drawing a spade from a deck of 52 playing cards?

a) $13/52$

b) 0.25

c) 25%

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

D. What is the probability of choosing a black colored pawn from a group of 4 black pawns and 6 white ones?

a) ~~4/10~~ $\frac{4}{10} = \frac{2}{5}$

b) ~~0.85~~ 0.4

c) ~~66.7%~~ 40%

E

E. What is the probability of rolling 2 dice the same in one roll of two dice?

a) $\frac{6}{36} = \frac{1}{6}$

b) $0.1\bar{6}$

c) 16.7%

E

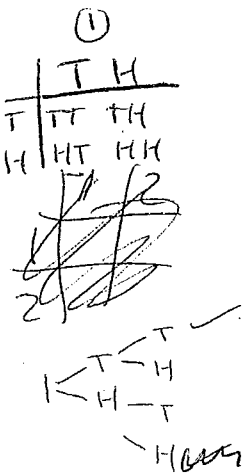
F. What is the probability of flipping two coins simultaneously and having them both come up heads?

a) ~~2/4~~ $\frac{1}{4}$

b) ~~0.8~~ 0.25

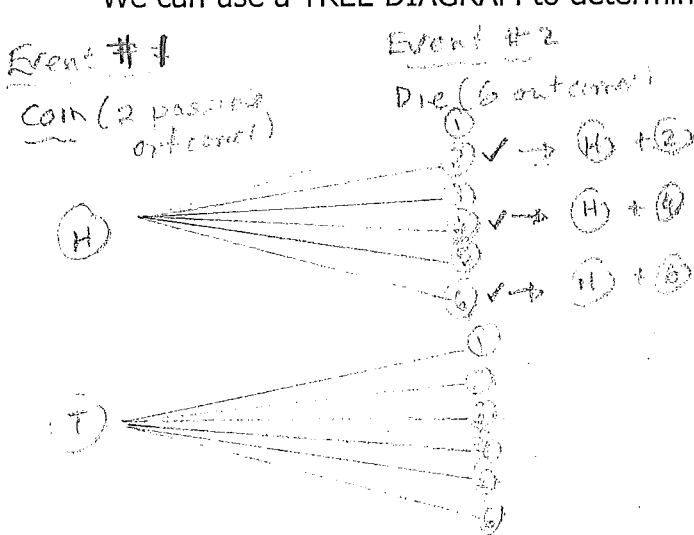
c) ~~80%~~ 25%

E



A coin is tossed and then, independently, a 'die' is thrown. What is the probability of a 'heads' turning up and an 'even number' being tossed?

We can use a TREE DIAGRAM to determine the probability.



- Total successful outcomes = 3
- Total possible outcomes = 12

$$\begin{aligned}
 \therefore P(\text{heads and even}) &= \frac{3}{12} \\
 &= \frac{1}{4} \\
 &= 0.25 \\
 &= 25\%
 \end{aligned}$$

How did we get a probability of : 1/4 or 0.25 or 25% ?

The probability of 2 or more independent events occurring can be determined by using the following formula.

The probability of events A and B occurring is:

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$\begin{aligned}
 P(\text{Heads and even}) &= P(\text{Heads}) \times P(\text{even}) \\
 &= \frac{1}{2} \times \frac{3}{6} \\
 &= \frac{3}{12} \\
 &= \frac{1}{4} \\
 &= 0.25 \\
 &= 25\%
 \end{aligned}$$

$$P(A \text{ and } B) = P(A) \times P(B)$$

* This formula only works if each event is INDEPENDENT of the other (one event does not affect the other).

1. A bag contains 5 black beads and 5 red beads. What is the probability of drawing 2 red beads if:

→ the two events are independent

a) The first bead is replaced before the 2nd bead is chosen

$$P(\text{red and red})$$

$$= P(\text{red}) \times P(\text{red})$$

$$= \frac{5}{10} \times \frac{5}{10}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4} \text{ or } 25\%$$

b) The first bead is NOT replaced before the 2nd bead is chosen

$$P(\text{red and red}) \neq \text{no replacement}$$

$$P(\text{red}) \times P(\text{red})$$

$$= \frac{5}{10} \times \frac{4}{9} \quad \begin{array}{l} \checkmark - 1 \text{ fewer red bead} \\ \checkmark - 1 \text{ fewer bead} \end{array}$$

$$= \frac{1}{2} \times \frac{4}{9}$$

$$= \frac{4}{18} = \frac{2}{9} \text{ or } 22\%$$

2. What is the probability of drawing an ace, from a deck of 52 cards, then replacing it and then drawing a red card?

$$P(\text{Ace and red})$$

$$= P(\text{ace}) \times P(\text{red})$$

$$= \frac{4}{52} \times \frac{26}{52}$$

$$= \frac{1}{13} \times \frac{1}{2}$$

$$= \frac{1}{26}$$

=

Events are independent if one outcome has no influence on the other outcome.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Find the probability of the following. Also: Show a tree diagram for Q # 1

1. Landing on heads after tossing a coin and rolling a 5 on a single 6-sided die

$$P(\text{heads}) = \frac{1}{2} \quad P(5) = \frac{1}{6}$$

$$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

2. Choosing 2 red marbles from a jar containing 3 red, 6 blue and 7 white and landing on heads after tossing a coin. (Not replacing)

$$P(\text{red}) = \frac{3}{16} \quad P(\text{2nd red}) = \frac{2}{15}$$

$$\rightarrow \frac{3}{16} \times \frac{2}{15} = \frac{1}{40} \quad \begin{array}{l} P(\text{heads}) \\ \downarrow \\ \frac{1}{2} \\ \hline \frac{1}{80} \end{array}$$

3. Choosing a 3 from a deck of cards, replacing it, then choosing an ace as the second card

$$P(3) = \frac{4}{52} = \frac{1}{13} \quad P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$$

$$\frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$

4. A dresser contains 1 pair of socks from each of the following colors: blue, brown, red, black, and white. Each pair is folded into matching pairs. You reach into the drawer and choose the red pair—the wrong color. You replace it and choose again. What is the probability that you choose red a second time?

$$P(\text{red}) = \frac{1}{5} \quad P(\text{red again}) = \frac{1}{5}$$

$$\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$$

5. A card is chosen from a deck of cards, then replaced. Then a second card is chosen. What is the probability of choosing a heart then a 4?

$$p(\text{heart}) = \frac{13}{52} = \frac{1}{4} \quad p(4) = \frac{4}{52} = \frac{1}{13}$$

$$\frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

6. From a jar containing 5 red balls, 4 green ones, and 1 yellow one. What is the probability of choosing a green ball, NOT replacing it, then choosing a red ball?

$$p(\text{green}) = \frac{4}{10} = \frac{2}{5} \quad p(\text{red}) = \frac{5}{9}$$

$$\rightarrow \frac{2}{5} \times \frac{5}{9} = \frac{2}{9}$$

7. A survey has determined that 70% of Claremont students like chocolate chip cookies. What is the probability that 3 students chosen at random would like chocolate chip cookies?

$$p(\text{ccc}) = \frac{70}{100} = \frac{7}{10}$$

$$p(\text{ccc}) = \frac{7}{10}$$

$$\therefore \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{343}{1000}$$

$$\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{343}{1000}$$

8. A pawn is chosen from the 16 white chess pieces, NOT replaced, then a second pawn being chosen.

$$p(\text{pawn}) = \frac{8}{16} = \frac{1}{2} \quad p(\text{2nd pawn}) = \frac{7}{15}$$

$$\frac{1}{2} \times \frac{7}{15} = \frac{7}{30}$$

9. Choosing an ACE from a deck of cards, replacing it, then choosing a red card

$$P(\text{ACE}) = \frac{4}{52} = \frac{1}{13} \quad P(\text{red}) = \frac{26}{52} = \frac{1}{2}$$

$$\frac{1}{13} \times \frac{1}{2} = \frac{1}{26}$$

10. Tossing a HEADS from a two-sided coin 5 times in a row

$$p(H) = \frac{1}{2} \quad p(H) = \frac{1}{2} \quad p(H) = \frac{1}{2} \quad p(H) = \frac{1}{2} \quad p(H) = \frac{1}{2}$$

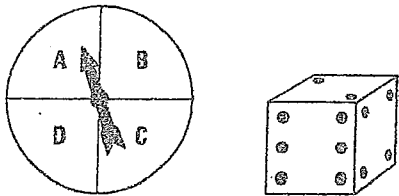
$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$$

9.5 Independent Events

MATHPOWER™ Nine, pp. 350-352

Events are said to be independent of each other if one outcome has no influence on the other outcome. For example, tossing a coin followed by rolling a die are independent events.

1. This spinner and die are used for an experiment.



Write the probability of each of the following.

a) spinning an A $\frac{1}{4}$ b) rolling a 2 $\frac{1}{6}$

c) spinning an A and rolling a 2 $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$

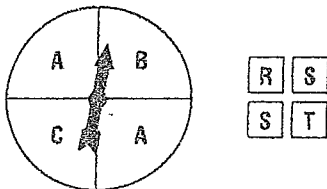
d) rolling a prime number and spinning a consonant $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

2. A yellow bag contains 1 red marble, 1 white marble, and 1 green marble. An orange bag contains 1 white marble and 1 black marble. If one marble is drawn from each bag, what is the probability of drawing each of the following?

a) a red marble and a black marble $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

b) a white marble from each bag $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

3. This spinner and these cards are used for an experiment.



What is the probability of each of the following?

a) spinning a B and choosing a T $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

b) spinning an A and choosing an S $\frac{1}{2} \times \frac{2}{4} = \frac{1}{4}$

c) spinning a consonant and choosing a consonant $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

4. A dime is tossed and a die is rolled. For each of the following, circle the option that has the higher probability.

a) tossing a head and rolling an even number $\frac{1}{2}$
OR
tossing a tail and rolling a number greater than 2 $\frac{1}{2}$

b) tossing a tail and rolling a prime number $\frac{1}{2}$
OR
tossing a head and rolling a number divisible by 3 $\frac{1}{2}$

5. Find the following probabilities for a true-or-false quiz.

a) the probability of guessing a correct answer $\frac{1}{2}$

b) the probability of guessing 2 correct answers in a row $\frac{1}{4}$

c) the probability of guessing 3 correct answers in a row $\frac{1}{8}$

6. Two dice are rolled. What is the probability of each event?

a) The numbers total 9. $\frac{4}{36} = \frac{1}{9}$

b) The numbers total 2. $\frac{1}{36}$

c) Both dice display the same number. $\frac{6}{36} = \frac{1}{6}$

d) The sum of the numbers is not 8. $\frac{29}{36}$