Physics 12
Gravitational Potential Energy Revisited and Satellites

1. From Physics 11 we have $E_p = mgh$. The E_p depends on h and g, but what is h and what is the g value there? Where do you start your measurement?

2. We need an Ep formula that will allow us to calculate at any location in space.

3. If we define the gravitational potential energy to be 0 at infinity and negative as we get closer to the earth then we have a more useful formula.

$$E_p = \frac{-Gm_1m_2}{r}$$

4. This new form of Ep allows us to calculate the total mechanical energy of any orbiting body. The total mechanical energy is:

$$TME = KE + PE$$

Or

$$TME = E_k + E_p$$

This is known as the law of conservation of mechanical energy

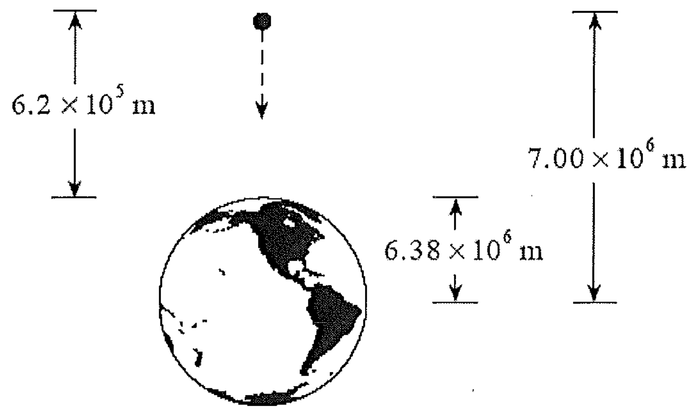
Example:

A satellite orbits the earth with a kinetic energy of 2.0×10^{10} J. Its gravitational potential energy in this orbit is -4.0×10^{10} J. What is the total energy of the satellite?

Total mechanical energy of the satellite is $E_k + E_p$.

$$2.0 \times 10^{10} J + -4.0 \times 10^{10} J = -2 \times 10^{10} J$$

Example: A 450 kg piece of space debris initially at rest falls from an altitude of 6.2×10^5 m above the earth's surface. What is its kinetic energy just before impact with the surface? (Ignore air resistance.)



$$\Delta Ep + \Delta Ek = 0$$

$$E_{p2} - E_{p1} + E_{k2} - E_{k1} = 0$$

$$E_{k2} = E_{k1} + E_{p1} - E_{p2}$$

$$E_{k2} = 0 + \frac{-Gm_1m_2}{r_1} - \frac{-Gm_1m_2}{r_2}$$

$$E_{k2} = \frac{-Gm_1m_2}{r_1} + \frac{Gm_1m_2}{r_2}$$

$$E_{k2} = \frac{-6.67 \times 10^{-11} \times 450 \times 5.98 \times 10^{24}}{7 \times 10^6} + \frac{6.67 \times 10^{-11} \times 450 \times 5.98 \times 10^{24}}{6.38 \times 10^6}$$

$$E_{k2} = 2.50 \times 10^9 \text{ J}$$

Total mechanical energy of the satellite is $E_k + E_p$.

$$TME = E_k + E_p$$

Since these objects are orbiting $F_c = F_g$

$$\frac{mv^2}{r} = \frac{Gm_1m_2}{r^2}$$

$$v^2 = \frac{Gm}{r}$$

If you substitute this value in the E_k portion of the TME formula you get:

$$TME = E_k + E_p$$

$$TME = \frac{1}{2} mv^2 + \frac{-Gm_1m_2}{r}$$

$$TME = \frac{Gm_1m_2}{2r} + \frac{-Gm_1m_2}{r}$$

$$TME = \frac{-Gm_1m_2}{2r}$$

The total mechanical energy is $\frac{1}{2}$ PE

Energy of an Orbiting Satellite

Total energy = PE + KE

KE = -½PE

Total Energy = PE + -½ PE

Total Energy = ½ PE

Sweet!