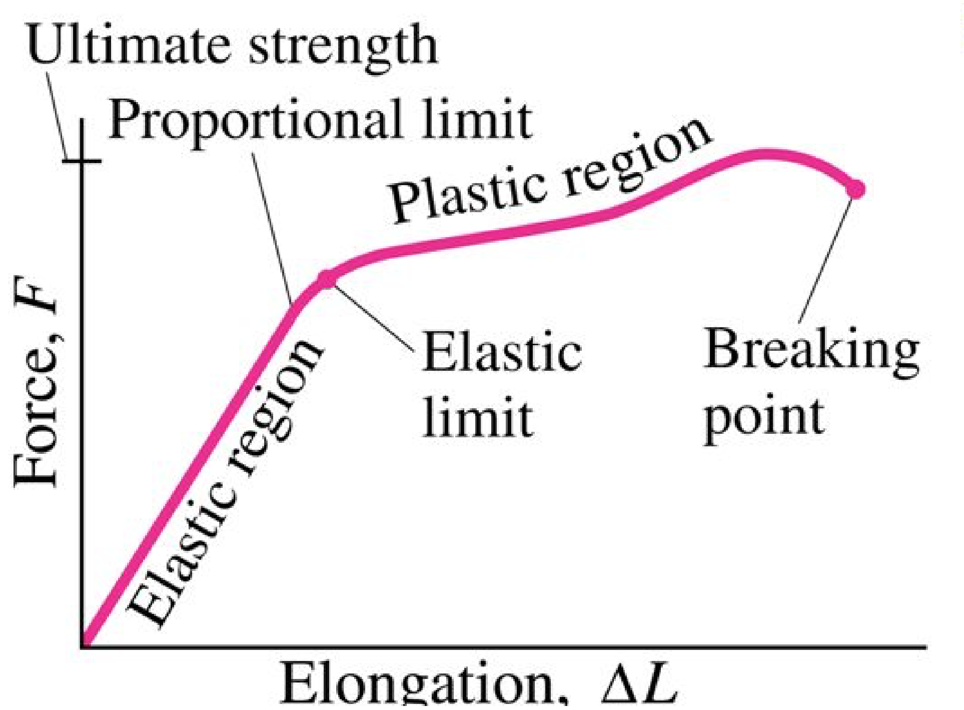
Lesson # 6 Stress and Strain

Strength of Materials

Stress and Strain

1. When forces are applied to an object, the object changes shape and if the forces are large enough the object will break or fracture.

2. As long as the object has not broken or fractured there is a mathematical relationship between the applied force and the change in shape of the object. This mathematical relationship was first proposed by Robert Hooke (1635-1703) and is commonly known as Hooke's Law $F = k\,\Delta L$. Where F is the applied force in Newton, k is the spring constant, and ΔL is the change in length of the object.



3. The above relationship is said to be linear up to the proportional limit. Beyond the proportional limit is the elastic limit which is the point that if the force is removed then the material will still return to its original length.

4. Beyond the elastic limit is the plastic region. The plastic region is the region where the material does not return to its original length once the force is removed.

5. And finally beyond the plastic region is the breaking point which is self explanatory.

6. The amount of elongation depends on the force applied, the material, and the dimensions of the material. Young's law can now be restated to include all three of the above mentioned values as:

$$\Delta L = \frac{1}{E}\frac{F L_o}{A}$$

F is the applied force (N)

L_o is the initial length (m)

E is the elastic modulus or Young's modulus (N/m^2) see table below.

Young's Modulus, Bulk Modulus, and Shear Modulus of Various Materials

Material	Young's Modulus ($10^9\ N/m^2$)	Bulk Modulus ($10^9\ N/m^2$)	Shear Modulus ($10^9\ N/m^2$)
aluminum	70	70	25
brass	100	80	40
concrete	30	13	15
iron	190	70	65
nylon	3	—	4.1
rubber band	0.005	—	0.003
steel	200	140	78
air	—	$1.01 \times 10^5\ N/m^2$	—
ethyl alcohol	—	1.0	—
water	—	2.0	—
human ACL	0.1	—	—
human lung	—	$1.5–9.8 \times 10^3\ N/m^2$	—
pig endothelial cell	—	—	$2 \times 10^4\ N/m^2$

7. The elastic modulus only depends on the material being stretched or compressed and not on the amount of the material, this makes it much more useful for calculations.

8. From the equation in point 6 we can see that the elongation is directly proportional to the product of the object's length and force per unit area.

9. Force per unit area is referred to as stress.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

10. Strain is defined to be the ratio of the change in length to the original length.

$$\text{Strain} = \frac{\Delta L}{L_o}$$

Example: A 1.60m steel piano wire has a diameter of 0.20cm. How great is the tension in the wire if it stretches 0.30cm when tightened?

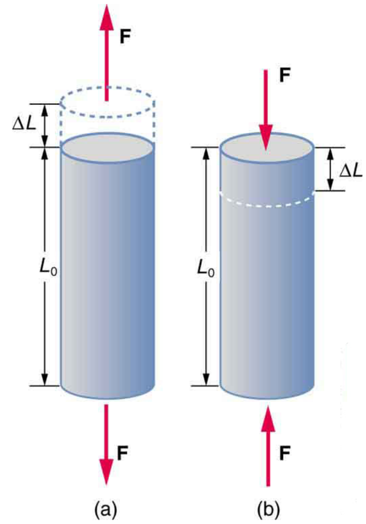
$$\Delta L = \frac{F\,L_o}{E\;A}$$

$$F = \frac{E\,\Delta L\,A}{L_o}$$

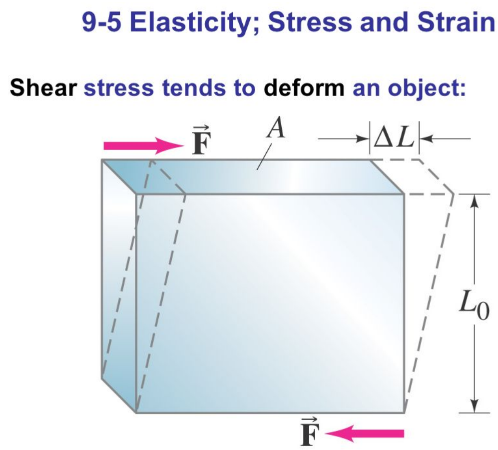
$$F = \frac{(2.0 \times 10^{11} N/m^2) \times 0.0030m \times (3.1 \times 10^{-6}\ m^2)}{1.60m}$$

$$F = 1200N$$

11. When a material is under tension it is said to be under tensile stress (left diagram). The opposite of tensile stress is compression stress (right diagram).



12. There are three types of stress, the above 2 and shear stress. With shear stress the object has equal and opposite forces acting on opposite faces.



13. The object dimensions do not significantly change but the shape of the object does change. An equation similar to the equation in number 6 can be stated for shear stress.

$$\Delta L = \frac{F L_o}{G A}$$

ΔL, L_o, F, A are indicated in the above diagram

G is the shear modulus (see the table in number 6).

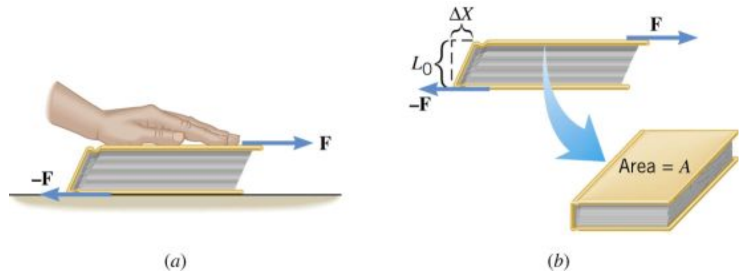
Example: A person applies a 10 N horizontal force on 10cm^3 sponge. The sponge does not slide but does distort. Determine the amount of distortion of the sponge if the shear modulus for a sponge is 1000 N/m^2.

$$\Delta L = \frac{1}{G} \frac{F L_o}{A}$$

$$\Delta L = \frac{1 \times 10\text{N} \times .1\text{m}}{1000\text{N/m}^2 \times 0.01\text{m}^2}$$

0.010 m

14. The object in number 12 has additional forces acting on it due to how the force is applied. The hand pushing on the book also compresses the object.



15. If an object is subject to forces on all sides then the volume will decrease, this is most commonly seen for objects submerged in a liquid. The equation from number 6 can once again be modified to incorporate the change in volume of the object as follows:

$$\Delta V = \frac{-1F\,V_o}{B\,\Delta A}$$

$$\Delta V = \frac{-1\,F\,V_o}{B\,\Delta A}$$

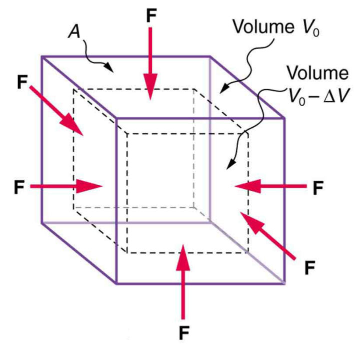
$$\frac{\Delta V}{V_o} = \frac{-1\,\Delta P}{B}$$

ΔP is the change in pressure

ΔV is the change in volume

B is the bulk modulus, see the table in number 6

The negative indicates that the object shrinks in size



Example: If a block of 1 m^3 concrete block is submerged in water to a depth that has a pressure of 1.1065×10^6 Pa (Pascals), then what will be the change in volume of the block? Surface pressure is 101325 Pa.

$$\frac{\Delta V}{V_o} = \frac{-1\ \Delta P}{B}$$

$$\Delta V = \frac{-1\ \Delta P\ V_o}{B}$$

$$\Delta V = \frac{-1\ (1.1065 \times 10^6\ \text{Pa} - 101325\ \text{Pa})\ 1\text{m}^3}{13 \times 10^9\ \text{N/m}^2}$$

$$\Delta V = -7.7 \times 10^{-5}\ \text{m}^3$$