Chapter 2 Review

Section 2.1 Extra Practice

1. For each polynomial function, state the degree. If the function is not a polynomial, explain why.

   a) $h(x) = 5 - \frac{1}{x}$ b) $y = 4x^2 - 3x + 8$

   c) $g(x) = -9x^6$ d) $f(x) = \sqrt[3]{x}$

2. What is the leading coefficient and constant term of each polynomial function?

   a) $f(x) = -x^3 + 2x + 3$ b) $y = 5 + 9x^4$

   c) $g(x) = 3x^4 + 3x^2 - 2x + 1$ d) $k(x) = 9 - 3x - 2x^2$

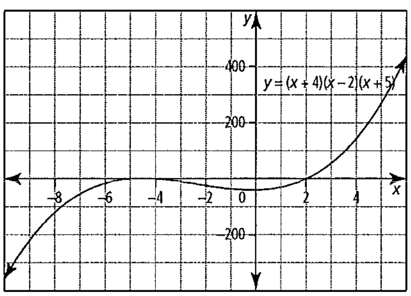
3. State whether the polynomial function is odd or even. Then, state whether the function has a maximum value, a minimum value, or neither.

   a) $g(x) = -x^3 + 8x^2 + 7x - 1$ b) $f(x) = x^4 + x^2 - x + 10$

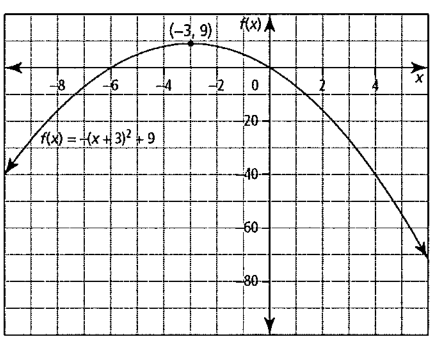
   c) $p(x) = -2x^5 + 5x^3 - 11x$ d) $h(x) = -3x^2 - 6x - 2$

4. State the number of real x-intercepts, domain, and range for each polynomial function.

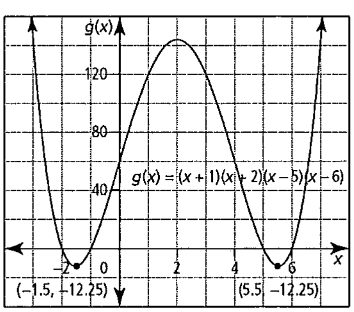
   a)



   b)



   c)



5. State the possible number of x-intercepts and the value of the y-intercept for each polynomial function.

   a) $f(x) = -x^3 + 2x + 3$ b) $y = 5 + 9x^4$

   c) $g(x) = 3x^4 + 3x^2 - 2x + 1$ d) $k(x) = -3x - 2x^2$

6. Identify the following characteristics for each polynomial function:
   - the type of function and whether it is of even or odd degree
   - the end behaviour of the graph of the function
   - the number of possible x-intercepts
   - whether the function will have a max or min value
   - the y-intercept

   a) $g(x) = -x^4 + 2x^2 + 7x - 5$ b) $f(x) = 2x^5 + 7x^3 + 12$

7. Given the polynomial $y = -2(x + 1)^2(x - 2)(x - 3)^2$, determine the following without graphing.

   a) Describe the end behaviour of the graph

   b) Determine the possible number of x-intercepts

   c) Determine the y-intercept of the function.

   d) Sketch the graph.

8. Identify each function as quadratic, cubic, quartic, or quintic.

   a) $y = -x^4 + 2x^2 + 7x - 5$ b) $f(x) = 2x^5 + 7x^3 + 12$

   c) $g(x) = -x^3 + 2x + 3$ d) $k(x) = 9 - 3x - 2x^2$

9. The height, h, in metres, above the ground of an object dropped from a height of 60 m is related to the length of time, t, in seconds, that the object has been falling. The formula is $h = -4.9t^2 + 60$.

   a) What is the degree of this function?

   b) What are the leading coefficient and constant of this function? What does the constant represent?

   c) What are the restrictions on the domain of the function? Explain why you selected those restrictions.

   d) Describe the end behaviour of the graph of this function.

10. Using the formula in #9, determine how long an object will take to hit the ground if it is dropped from a height of 60 m. Write your answer to the nearest tenth of a second.

Section 2.2 Extra Practice

1. Use long division to divide $x^2 - x - 15$ by $x - 4$.

   a) Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.

   b) Identify any restrictions on the variable.

   c) Write the corresponding statement that can be used to check the division.

   d) Verify your answer.

d) $-2x^2(x+3)(x+5)(x-7)$

2. Divide the polynomial $P(x) = x^4 - 3x^3 + 2x^2 + 55x - 11$ by $x + 3$.

   a) Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.

   b) Identify any restrictions on the variable.

   c) Verify your answer.

3. Determine each quotient using long division.

   a) $(3x^2 - 13x - 2) \div (x - 4)$ b) $\frac{2x^3 - 10x^2 - 15x - 20}{x+5}$

   c) $(2w^4 + 3w^3 - 5w^2 + 2w - 27) \div (w + 3)$

4.Determine each remainder using long division.

   a) $(3w^3 - 5w^2 + 2w - 27) \div (w - 5)$

   b) $\frac{2x^3 - 8x^2 - 5x - 2}{x+1}$ c) $(3x^2 - 13x - 2) \div (x + 2)$

5.Determine each quotient using synthetic division.

   a) $(4w^4 + 3w^3 - 7w^2 + 2w - 1) \div (w + 2)$

   b) $\frac{x^4 + 2x^3 - 8x^2 - 5x - 2}{x-2}$ c) $(5y^4 + 2y^2 - y + 4) \div (y + 1)$

6. Determine each remainder using synthetic division.

   a) $(3x^2 - 16x + 5) \div (x - 5)$

   b) $(2x^4 - 3x^3 - 5x^2 + 6x - 1) \div (x + 3)$

   c) $(4x^3 + 5x^2 - 7) \div (x - 2)$

7.Use the remainder theorem to determine the remainder when each polynomial is divided by $x + 2$.

   a) $-4x^4 - 3x^3 + 2x^2 - x + 5$

   b) $7x^5 + 5x^4 + 23x^2 + 8$ c) $8x^3 - 1$

8. Determine the remainder resulting from each division.

   a) $(3x^3 - 4x^2 + 6x - 9) \div (x + 1)$

   b) $(3x^2 - 8x + 4) \div (x - 2)$ c) $(6x^3 - 5x^2 - 7x + 9) \div (x + 5)$

9. For $(2x^3 + 5x^2 - kx + 9) \div (x + 3)$, determine the value of k if the remainder is 6.

10. When $4x^2 - 8x - 20$ is divided by $x + k$, the remainder is 12. Determine the value(s) of k.

Section 2.3 Extra Practice

1. What is the corresponding binomial factor of a polynomial $P(x)$ given the value of the zero?

   a) $P(6) = 0$ b) $P(-7) = 0$ c) $P(2) = 0$ d) $P(-5) = 0$

2. Determine whether $x - 1$ is a factor of each polynomial.

3. State whether each polynomial has $x + 2$ as a factor.

   a) $-3x^3 + 2x^2 + 10x + 5$ b) $5x^2 + 6x - 8$

   c) $2x^4 - 3x^3 - 5x^2$ d) $3x^3 - 12x - 2$

4. What are the possible integral zeros of each polynomial?

   a) $P(n) = n^3 - 2n^2 - 5n + 12$

   b) $P(p) = p^4 - 3p^3 - p^2 + 7p - 6$

   c) $P(z) = z^4 + 4z^3 + 3z^2 + 8z - 25$

   d) $P(y) = y^4 - 11y^3 - 2y^2 + 2y + 10$

5. The factors of a polynomial are $x + 3$, $x - 4$, and $x + 1$. Describe how the zeros of the polynomial expression could be used to determine the zeros of the corresponding function.

6.Factor completely.

   a) $x^3 + 2x^2 - 13x + 10$ b) $x^4 - 7x^3 + 3x^2 + 63x - 108$

   c) $x^3 - x^2 - 26x - 24$ d) $x^4 - 26x^2 + 25$

7. Factor completely.

   a) $x^3 + x^2 - 16x - 16$ b) $x^3 - 2x^2 - 6x - 8$

   c) $k^3 + 6k^2 - 7k - 60$ d) $x^3 - 27x + 10$

8. Factor completely. a) $x^4 + 4x^3 - 7x^2 - 34x - 24$

   b) $x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

9. Determine the value(s) of k so that the binomial is a factor of the polynomial.

   a) $x^2 - 8x - 20$, $x + k$ b) $x^2 - 3x - k$, $x - 7$

10. Each polynomial has a factor of $x - 3$. What is the value of k in each case?

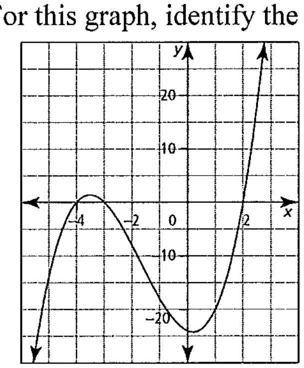
   a) $kx^3 - 10x^2 + 2x + 3$ b) $4x^4 - 3x^3 - 2x^2 + kx - 9$

Section 2.4 Extra Practice

1.Solve. a) $(x + 5)(x + 2)(x - 3)(x - 6) = 0$ b) $x^3 - 27 = 0$

   c) $(3x + 1)(x - 4)(x - 7) = 0$ d) $x(x + 4)^3(x + 2)^2 = 0$

2.For this graph, identify the following:



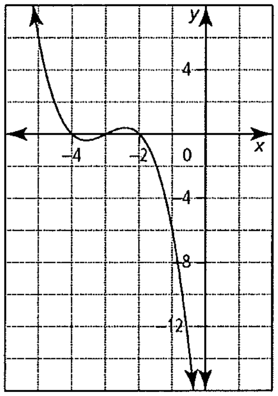
a) the zeros

b) the intervals where the function is positive

c) the intervals where the function is negative

a) $-4x^4 - 3x^3 + 2x^2 - x + 5$ **b)** $7x^5 + 5x^4 + 23x^2 + 8$
c) $2x^4 - 3x^3 - 5x^2 + 6x - 1$ **d)** $2x^3 + 5x^2 - 7$

3. For the graph of this polynomial function, determine



a) the least possible degree
b) the sign of the leading coefficient
c) the x-intercepts and the factors of the function
d) the intervals where the function is positive and the intervals where it is negative

4. The graph of $y = x^3$ is transformed to obtain the graph of $y = -2(4(x + 1))^3 - 5$. Copy and complete the table.

$y = x^3$	$y = (4x)^3$	$y = -2(4x)^3$	$y = -2(4(x + 1))^3 - 5$
(–2, –8)			
(–1, –1)			
(0, 0)			
(1, 1)			
(2, 8)			

5. The graph of $y = x^4$ is transformed to obtain the graph of $y = \frac{1}{4}\left(\frac{1}{2}(x-9)\right)^4 + 3$. Copy and complete the table.

$y = x^4$	$y = \left(\frac{1}{2}x\right)^4$	$y = \frac{1}{4}\left(\frac{1}{2}x\right)^4$	$y = \frac{1}{4}\left(\frac{1}{2}(x-9)\right)^4 + 3$
(–2, –16)			
(–1, 1)			
(0, 0)			
(1, 1)			
(2, 16)			

6. For the graph of this polynomial function, determine the following:

a) the least possible degree
b) the sign of the leading coefficient
c) the x-intercepts and the factors of the function
d) the intervals where the function is positive and the intervals where it is negative

7. Without using a graphing calculator, determine the following for $y = x^3 + 4x^2 - x - 4$:

a) the zeros of the function
b) the degree and end behaviour of the function
c) the y-intercept
d) the intervals where the function is positive and the intervals where it is negative

8. Sketch a graph of each function without using technology. Label all intercepts.

a) $y = x^3 - 4x^2 - 5x$ **b)** $f(x) = -x^4 + 19x^2 + 6x - 72$
c) $g(x) = x^5 - 14x^4 + 69x^3 - 140x^2 + 100x$

9. Determine the equation with least degree for each polynomial function.

a) a cubic function with zeros 3 (multiplicity 2) and –1, and y-intercept = 18
b) a quintic function with zeros –2 (multiplicity 3) and 4 (multiplicity 2), and y-intercept = –32
c) a quartic function with zeros –1 (multiplicity 2) and 5 (multiplicity 2), and y-intercept = –10

10. Determine three consecutive integers with a product of –504.

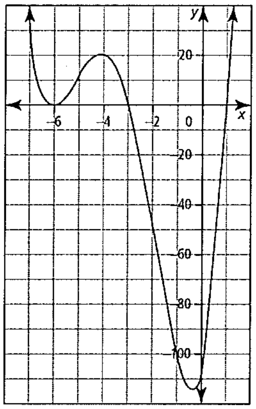
11. A toothpaste box has square ends. The length of the box is 12 cm greater than the width. The volume is 135 cm^3. What are the dimensions of the box?

12. The dimensions of a rectangular prism are 10 cm by 10 cm by 5 cm. When each dimension is increased by the same length, the new volume is 1008 cm^3. What are the dimensions of the new prism?

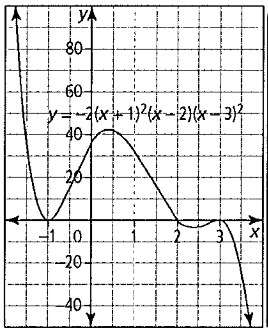
Answers

Section 2.1 Extra Practice

1. a) Not a polynomial; the exponent of the variable is not a whole number: $\frac{1}{x} = x^{-1}$ **b)** degree = 2 **c)** degree = 6
d) Not a polynomial; the exponent of the variable is not a whole number: $\sqrt[3]{x} = x^{\frac{1}{3}}$ **2. a)** –1; 3 **b)** 9; 5 **c)** 3; 1 **d)** –2; 9
3. a) odd; neither **b)** even; min **c)** odd; neither **d)** even; max
4. a) 3; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$
b) 2; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \le 9, y \in R\}$
c) 4; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \ge -12.25, y \in R\}$
d) 4; domain: $\{x \mid x \in R\}$; range: $\{y \mid x \in R\}$
5. a) 0, 1, 2, or 3; y-intercept = 3 **b)** 0, 1, 2, 3, or 4; y-intercept = 5
c) 0, 1, 2, 3, or 4; y-intercept = 1 **d)** 0, 1, or 2; y-intercept = 0
6. a) degree of 4, even-degree polynomial; opens downward, extends down into quadrant III and down into quadrant IV; maximum of four x-intercepts; has a maximum value; y-int = 5
b) degree of 5, odd-degree polynomial; extends up into quadrant I and down into quadrant III; maximum of 5 x-intercepts; no maximum or minimum values; y-intercept = 12
d) positive intervals: $x < -4$ and $-3 < x < -2$ negative interval: $-4 < x$



7. a) extends up into quadrant II and down into quadrant IV
b) 3 **c)** 36
d)



8. a) quartic **b)** quintic **c)** cubic **d)** quadratic

9. a) 2 **b)** −4.9; constant = 60; The constant represents the height the object fell from.

c) The domain, t, must be greater than or equal to zero, because it represents time.

9 d) opens downward; lies only within q. I; points begin on the y-axis and end on the x-axis; maximum value = 60 **10.** 3.5 s

Section 2.2 Extra Practice

1. a) $\frac{x^2 - x - 15}{x - 4} = (x + 3) - \frac{3}{x - 4}$ **b)** $x \neq 4$

c) $x^2 - x - 15 = (x - 4)(x + 3) - 3$

d) To check, multiply the divisor by the quotient and add the remainder.

2. a) $\frac{x^4 - 3x^3 + 2x^2 + 55x - 11}{x + 3} = (x^3 - 6x^2 + 20x - 5) + \frac{4}{x + 3}$

b) $x \neq -3$ **c)** To check, multiply the divisor by the quotient and add the remainder.

3. a) $3x + 1$ **b)** $2x^2 - 20x + 85$ **c)** $2w^3 - 3w^2 + 4w - 10$

4. a) 233 **b)** −7 **c)** 36

5. a) $4w^3 - 5w^2 + 3w - 4$ **b)** $x^3 + 4x^2 - 5$ **c)** $5y^3 - 5y^2 + 7y - 8$

6. a) 0 **b)** 179 **c)** 45 **7. a)** −25 **b)** −44 **c)** −65

8. a) −22 **b)** 0 **c)** 831 **9.** 2 **10.** 4 and −2

Section 2.3 Extra Practice

1. a) $x - 6$ **b)** $x + 7$ **c)** $x - 2$ **d)** $x + 5$

2. a) No **b)** No **c)** No **d)** Yes **3. a)** No **b)** Yes **c)** No **d)** No

4. a) ±1, ±2, ±3, ±4, ±6, ±12 **b)** ±1, ±2, ±3, ±6
c) ±1, ±5, ±25 **d)** ±1, ±2, ±5, ±10

5. Example: Since the factors are $x + 3$, $x - 4$, and $x + 1$, the corresponding zeros of the function are −3, 4, and −1. The zeros can be confirmed by graphing $P(x)$ and using the trace or zero feature of a graphing calculator.

6. a) $(x - 1)(x - 2)(x + 5)$ **b)** $(x - 3)^2(x + 3)(x - 4)$
c) $(x + 1)(x - 4)(x - 6)$ **d)** $(x - 1)(x + 1)(x - 5)(x + 5)$

< −3 and $x > -2$

4.

$y = x^3$	$y = (4x)^3$	$y = -2(4x)^3$	$y = -2(4(x + 1))^3 - 5$
(−2, −8)	(−0.5, −8)	(−0.5, 16)	(−1.5, 11)
(−1, −1)	(−0.25, −1)	(−0.25, 2)	(−1.25, −3)
(0, 0)	(0, 0)	(0, 0)	(−1, −5)
(1, 1)	(0.25, 1)	(0.25, −2)	(−0.75, −7)
(2, 8)	(0.5, 8)	(0.5, −16)	(−0.5, −21)

5.

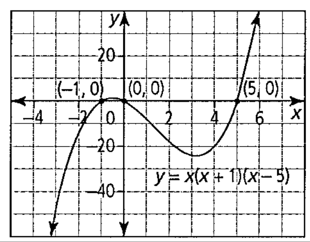
$y = x^4$	$y = \left(\frac{1}{2}x\right)^4$	$y = \frac{1}{4}\left(\frac{1}{2}x\right)^4$	$y = \frac{1}{4}\left(\frac{1}{2}(x - 9)\right)^4 + 3$
(−2, −16)	(−4, −16)	(−4, −4)	(5, −1)
(−1, 1)	(−2, 1)	(−2, 0.25)	(7, 3.25)
(0, 0)	(0, 0)	(0, 0)	(9, 3)
(1, 1)	(2, 1)	(2, 0.25)	(11, 3.25)
(2, 16)	(4, 16)	(4, 4)	(13, 7)

6. a) 4 **b)** positive **c)** −6, −3, 1; $(x + 6)$, $(x + 3)$, $(x - 1)$

d) positive intervals: (−∞, −6), (−6, −3), (1, ∞); negative interval: (−3, 1)

7. a) −4, −1, 1 **b)** 3; starts in quadrant III and extends to quadrant I
c) −4 **d)** positive intervals: (−4, −1), (1, ∞); negative intervals: (−∞, −4), (−1, 1)

8. a)



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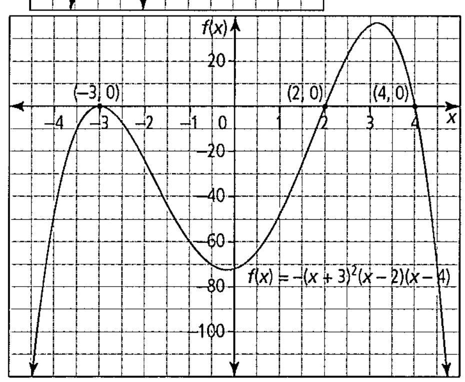

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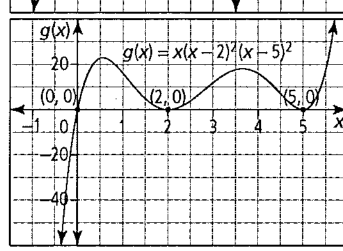

9. a) $y = 2(x - 3)^2(x + 1)$

b) $y = -\frac{1}{4}(x + 2)^3(x - 4)^2$

c) $f(x) = -\frac{2}{5}(x + 1)^2(x - 5)^2$

10. −9, −8, −7 **11.** 3 cm by 3 cm by 15 cm

12. 12 cm by 12 cm by 7 cm

7. a) $(x + 1)(x - 4)(x + 4)$ **b)** $(x - 4)(x^2 + 2x + 2)$
c) $(k - 3)(k + 4)(k + 5)$ **d)** $(x - 5)(x^2 + 5x - 2)$

8. a) $(x + 4)(x + 2)(x + 1)(x - 3)$ **b)** $(x + 3)(x + 2)(x + 1)(x - 1)(x - 2)$

9. a) 2, −10 **b)** 28 **10. a)** 3 **b)** −72

Section 2.4 Extra Practice

1. a) −5, −2, 3, 6 **b)** ±3 **c)** $-\frac{1}{3}$, 4, 7 **d)** 0, −2, −4 **2. a)** −3, 2, −4
b) (−4, −3), (2, ∞) **c)** (−∞, −4), (−3, 2) **3. a)** 3 **b)** negative
c) −4, −2, −3; $(x + 4)$, $(x + 2)$, $(x + 3)$