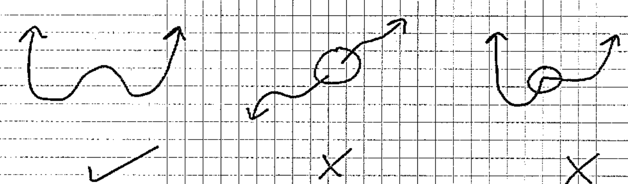
2.1 POLYNOMIALS

The degree of a function is the exponent of the highest power of x in the equation.

Types of functions:

CONSTANT	$f(x) = 7$	Degree: 0	
LINEAR	$f(x) = 3x+1$	Degree: 1	
QUADRATIC	$f(x) = 2x^2+x-3$	Degree: 2	
CUBIC	$f(x) = x^3+7$	Degree: 3	
QUARTIC	$f(x) = x^4+x^2$	Degree: 4	$y = x^3y + x^2$
QUINTIC	$f(x) = x^5+3x^4$	Degree: 5	$y = x^2y^3 + x$

Polynomial functions are always continuous
- no breaks or sharp corners
- can draw w/o lifting pencil



✓ ✗ ✗

NOT POLYNOMIAL FUNCTIONS!

$y = \sqrt{x+6} \rightarrow \sqrt{x}$ $\qquad$ $y = \sqrt{5}\,x$ ✓

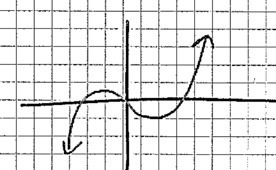
$y = \dfrac{4x^2-7}{3x} \rightarrow \dfrac{1}{3x}$ $\qquad$ $y = \dfrac{2x+7}{3}$ ✓

$y = 5x^2 + 2^x \rightarrow 2^{x}$

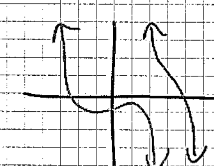
$y = \sqrt{-2}\,x$

Odd Degree Functions - linear, cubic ...

Positive Leading Coefficient		Negative Leading Coefficient
- rises to the right	← END BEHAVIOUR →	- falls to the right
Ex $y = x+2$ OR $y = 3x^3 - 5x$		Ex $y = -3x$, $y = -2x^3$



III → I
"starts down and ends up"



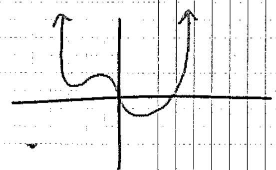
II → IV
"starts up and ends down"

Even Degree Functions - quadratic, quartic

Positive Leading Coefficient — END BEHAVIOUR — **Negative Leading Coefficient**

- opens up / rises left and right

Ex $y = 3x^2 - 1$, $y = 5x^4 - x^3$

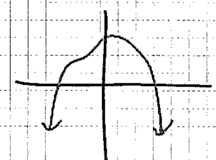


II → I

"starts up and ends up"

- opens down / falls left and right

Ex $y = -x^2$, $y = -3x^4 + 7$



III → IV

"starts down and ends down"

To determine the y-intercept → $x = 0$

Ex $y = 3(x-2)^2$

$y = 3(-2)^2$

$y = 12$

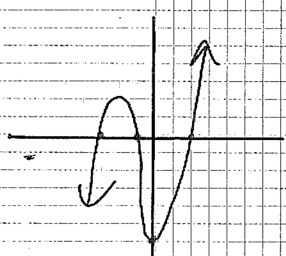
$f(x) = (x+2)(2x-1)$

$f(0) = (2)(-1)$

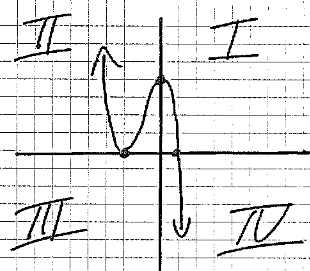
$f(0) = -2$

① Graph: (Determine y-intercept, zeros)

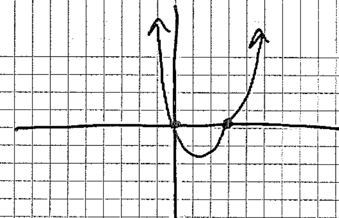
a) $y = (x+1)(x-2)(x+3)$



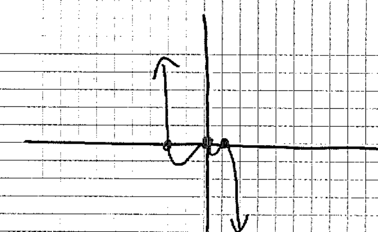
b) $y = -(x-1)(x+2)^2$



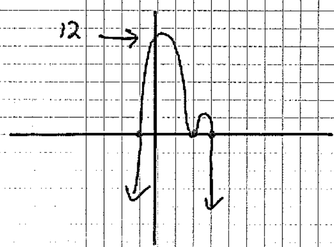
c) $y = x(x-3)^3$



d) $y = -x^2(x-1)^2(x+2)$



e.) $y = -(x-2)^2(x+1)(x-3)$



y int: $-(-2)^2(1)(-3)$

$-(4)(-3)$

$+12$

② Determine the zeros

a) $y = -3x^4 + 3x^2$

$y = -3x^2(x^2-1)$

$y = -3x^2(x-1)(x+1)$

$\boxed{x = 0 \text{ "of multiplicity 2"} \quad x = +1, -1}$

b) $y = x^3 - 3x^2 - 4x + 12$

$y = x^2(x-3) - 4(x-3)$

$y = (x^2-4)(x-3)$

$y = (x+2)(x-2)(x-3)$

$\boxed{x = \pm 2, 3}$

Pg 73 #2-4, 5a), 6, 7, 9

Note Title

2.3 Division of Polynomials - Long Division

9/19/2012

Recall: Long Division

$784 \div 17 = ?$

$$
\begin{array}{r}
46 \\
17 \overline{)\,784} \\
-68\downarrow \\
\hline
104 \\
-102 \\
\hline
2
\end{array}
$$

DIVISOR: 17

QUOTIENT: 46

DIVIDEND: 784

REMAINDER: 2

① $(3x^2+8x+11) \div (x+2)$

$$
\begin{array}{r}
3x+2 \\
x+2\,\overline{)\,3x^2+8x+11} \\
-(3x^2+6x)\downarrow \\
\hline
2x+11 \\
-(2x+4) \\
\hline
7
\end{array}
$$

* Two ways to write answer:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}} \quad \text{OR} \quad \text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

$$\frac{3x^2+8x+11}{x+2} = 3x+2+\frac{7}{x+2} \quad \text{OR} \quad 3x^2+8x+11 = (3x+2)(x+2)+7$$

② Divide x^3+2x^2-5x-6 by $x-2$

$$
\begin{array}{r}
x^2+4x+3 \\
x-2\,\overline{)\,x^3+2x^2-5x-6} \\
-(x^3-2x^2)\downarrow \\
\hline
4x^2-5x \\
-(4x^2-8x)\downarrow \\
\hline
3x-6 \\
-(3x-6) \\
\hline
0
\end{array}
$$

$$\frac{x^3+2x^2-5x-6}{x-2} = x^2+4x+3$$

OR

$$x^3+2x^2-5x-6 = (x^2+4x+3)(x-2)$$

$$\frac{x^3+2x^2-5x-6}{x-2} = x^2+4x+3$$

OR

$$x^3+2x^2-5x-6 = (x^2+4x+3)(x-2)$$

③ $(6x^3+2x-1)\div(2x-4)$

$$\begin{array}{r} 3x^2+6x+13 \\ 2x-4\,\overline{)\,6x^3+0x^2+2x-1} \\ -(6x^3-12x^2) \\ \hline 12x^2+2x \\ -(12x^2-24x) \\ \hline 26x-1 \\ -(26x-52) \\ \hline 51 \end{array}$$

$$\frac{6x^3+2x-1}{2x-4} = 3x^2+6x+13+\frac{51}{2x-4}$$ OR $$6x^3+2x-1=(3x^2+6x+13)(2x-4)+51$$

④ $(4x^3+5x-3)\div(x+2)$

$$\begin{array}{r} 4x^2-8x+21 \\ x+2\,\overline{)\,4x^3+0x^2+5x-3} \\ -(4x^3+8x^2) \\ \hline -8x^2+5x \\ -(-8x^2-16x) \\ \hline 21x-3 \\ -(21x+42) \\ \hline -45 \end{array}$$

$$\frac{4x^3+5x-3}{x+2} = 4x^2-8x+21-\frac{45}{x+2}$$

OR

$$4x^3+5x-3=(4x^2-8x+21)(x+2)-45$$

5 $(2x^3+4x^2-2x+6) \div (2x^2+1)$

$$
\begin{array}{r}
x+2 \\
2x^2+0x+1 \overline{)\,2x^3+4x^2-2x+6} \\
-(2x^3+0x^2+x) \downarrow \\
\hline
4x^2-3x+6 \\
-(4x^2+0x+2) \\
\hline
-3x+4
\end{array}
$$

Pg 89

#1 (a, c, e)

$$\frac{2x^3+4x^2-2x+6}{2x^2+1} = x+2 - \frac{3x+4}{2x^2+1} \quad \text{OR} \quad 2x^3+4x^2-2x+6 = (x+2)(2x^2+1) - 3x+4$$

Note Title 9/20/2012

2.3 Division of Polynomials - Synthetic Division

Faster method of division → only work with the coefficients

Ex Divide $3x^2+8x+11$ by $x+2$

$$\begin{array}{r|rrr} -2 & 3 & 8 & 11 \\ & \downarrow & -6 & -4 \\ \hline & 3 & 2 & \boxed{7} \end{array}$$

$3x+2$ R 7

$$\frac{3x^2+8x+11}{x+2} = 3x+2+\frac{7}{x+2}$$

① Divide

a) $x^3-7x+4x^2-10 \div x+5$ * Re-arrange in descending order

$$\begin{array}{r|rrrr} -5 & 1 & 4 & -7 & -10 \\ & \downarrow & -5 & 5 & 10 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

x^2-x-2 R 0

$$\frac{x^3+4x^2-7x-10}{x+5} = x^2-x-2+\frac{0}{x+5}$$

$$x^3+4x^2-7x-10 = (x^2-x-2)(x+5)$$

b) $x^3-7x+6 \div x-3$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & 6 \\ & \downarrow & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & \boxed{12} \end{array}$$

$$\frac{x^3-7x+6}{x-3} = x^2+3x+2+\frac{12}{x-3}$$ OR

$$x^3-7x+6 = (x^2+3x+2)(x-3)+12$$

c) $4x^3-15x+2 \div x-3$

$$\begin{array}{r|rrrr} 3 & 4 & 0 & -15 & 2 \\ & \downarrow & 12 & 36 & 63 \\ \hline & 4 & 12 & 21 & \boxed{65} \end{array}$$

$$\frac{4x^3-15x+2}{x-3} = 4x^2+12x+21+\frac{65}{x-3}$$

OR

$$4x^3-15x+2 = (4x^2+12x+21)(x-3)+65$$

d) $4x^4-8x^3+11x^2+1$ by $2x-1$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 4 & -8 & 11 & 0 & 1 \\ & \downarrow & 2 & -3 & 4 & 2 \\ \hline & 4 & -6 & 8 & 4 & \boxed{3} \end{array}$$

* The coefficient of x must be 1

$2x-1=0$
$2x=1$
$x=\frac{1}{2}$

$$\frac{4x^4-8x^3+11x^2+1}{2x-1} = 2x^3-3x^2+4x+2+\frac{3}{2x-1}$$

$\div 2$

OR

$$4x^4-8x^3+11x^2+1 = (2x^3-3x^2+4x+2)(2x-1)+3$$

(2) When x^3+kx+1 is divided by $x-2$ the remainder is -3. Determine k.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & k & 1 \\ & \downarrow & 2 & 4 & 2k+8 \\ \hline & 1 & 2 & k+4 & \boxed{-3} \end{array}$$

$\therefore 1+2k+8=-3$
$2k=-12$
$\boxed{k=-6}$

Pg 90 #2 (a, c, g)
3 (a, c, d)
4 (c)

Note Title 9/23/2012

2.4 The Remainder + Factor Theorems

① a) Determine the remainder given $(x^3+4x^2+x-2)\div(x-2)$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & 1 & -2 \\ & \downarrow & 2 & 12 & 26 \\ \hline & 1 & 6 & 13 & \boxed{24} \end{array}$$

Remainder = 24

b) What is the value of $f(2)$ if $f(x)=x^3+4x^2+x-2$?

$$f(2) = 2^3+4(2)^2+2-2$$
$$= 8+16$$
$$= 24$$ Same as Remainder

* When a polynomial $P(x)$ is divided by $x-a$, the remainder is $P(a)$.

"Remainder Theorem"

② Determine the remainder when x^3-4x^2+5x+1 is divided by:

a) $x-2$

$$f(2)=2^3-4(2)^2+5(2)+1$$
$$=8-16+10+1$$
$$=\boxed{3}$$

b) $x+1$

$$f(-1)=(-1)^3-4(-1)^2+5(-1)+1$$
$$=-1-4-5+1$$
$$=\boxed{-9}$$

③ When $x^3+3x^2+cx+10$ is divided by $x-2$, the remainder is 6. Determine c.

$$2^3+3(2)^2+c(2)+10=6$$
$$8+12+2c+10=6$$
$$2c=6-30$$
$$2c=-24$$
$$\boxed{c=-12}$$

When a polynomial $P(x)$ is divided by $x - a$ and the remainder of $P(a) = 0$, $x - a$ is a factor of $P(x)$.

"FACTOR THEOREM"

④ Given $f(x) = x^3 - 3x^2 - 6x + 8$, which is a factor of $f(x)$?

a) $x + 3$

$$f(-3) = (-3)^3 - 3(-3)^2 - 6(-3) + 8$$
$$= -27 - 27 + 18 + 8$$
$$= -28$$

$x+3$ is NOT a factor

b) $x - 4$

$$f(4) = 4^3 - 3(4)^2 - 6(4) + 8$$
$$= 64 - 48 - 24 + 8$$
$$= 0 \checkmark$$

$x-4$ is a factor

RATIONAL ROOT THEOREM

$$\text{Possible Factors} = \frac{\text{Factors of Constant Term}}{\text{Factors of Leading Coefficient}}$$

Ex $3x^3 - 2x^2 + x - 4$

FACTOR OPTIONS: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} = \boxed{\pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}}$

⑤ Factor using the Factor / Rational Root Theorem

a) $f(x) = 2x^3 + 7x^2 + 2x - 3$

Options $= \dfrac{\text{Factors of } 3}{\text{Factors of } 2} = \dfrac{\pm 1, \pm 3}{\pm 1, \pm 2} = \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

$2x^2 + 5x - 3$
$2x^2 + 6x - x - 3$
$2x(x+3) - 1(x+3)$
$(2x-1)(x+3)$

$f(-1) = 0 \quad \therefore (x+1)$ is a factor

$$\begin{array}{r|rrrr} -1 & 2 & 7 & 2 & -3 \\ & \downarrow & -2 & -5 & 3 \\ \hline & 2 & 5 & -3 & \boxed{0} \end{array}$$

$(x+1)(2x^2+5x-3)$

$$\boxed{(x+1)(2x-1)(x+3)}$$

b) $f(x) = 4x^3 - 8x^2 - x + 2$

Options: $\dfrac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4} = \pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$

$f(2) = 0 \quad \therefore x - 2$ is a factor

$$\begin{array}{r|rrrr} 2 & 4 & -8 & -1 & 2 \\ & \downarrow & 8 & 0 & -2 \\ \hline & 4 & 0 & -1 & \boxed{0} \end{array}$$

$4x^2 - 1$

$$\boxed{(2x-1)(2x+1)(x-2)}$$

Pg 97

\# 1, 2(a), 3b), 4(a, c, e), 5e)

2 g) $1^4 + k(1)^3 - m(1) + 15 = 0$

$1 + k - m + 15 = 0$

$k - m = -16$

$\boxed{k + 16 = m}$ * 1

$6 + 16 = m$

$\boxed{m = 22}$

$(-3)^4 + k(-3)^3 - m(-3) + 15 = 0$

$81 - 27k + 3m + 15 = 0$

$-27k + 3m = -96$

$\div 3$ $\boxed{-9k + m = -32}$ * 2

sub 1 into 2

$-9k + k + 16 = -32$

$-8k = -48$

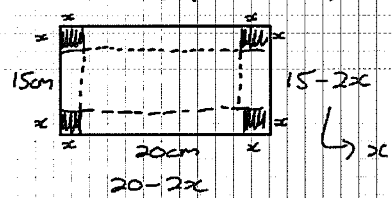
$\boxed{k = 6}$

Note Title

2.5 Polynomial Applications

9/24/2012

① An open top rectangular box is constructed by cutting a square of length x from each corner of a 20cm $\times$ 15cm rectangle, and then folding up the sides. What is the size of square cut out if the volume is $250cm^3$? ($x \geq 2cm$) Dimensions?



$V = (20-2x)(15-2x)(x)$

$250 = (20-2x)(15-2x)(x)$

$\hookrightarrow x < 7.5$

$V = \text{length} \times \text{width} \times \text{height}$

$250 = (20-2x)(15-2x)(x)$

$250 = (300 - 70x + 4x^2)(x)$

$250 = 300x - 70x^2 + 4x^3$

$0 = 4x^3 - 70x^2 + 300x - 250 \quad (\div 2)$

$0 = 2x^3 - 35x^2 + 150x - 125$

Options: $\dfrac{1, 5, 25, 125}{1, 2} \rightarrow \underbrace{5, \tfrac{5}{2}}_{x \geq 2,\ x < 7.5}$ * No ±

Try 5:

5	2	-35	150	-125
	↓	10	-125	125
	2	-25	25	0 ✓

Solve $2x^2 - 25x + 25$ using quadratic formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{25 \pm \sqrt{625 - 4(2)(25)}}{4} = \dfrac{25 \pm \sqrt{425}}{4} = \dfrac{25 \pm 20.62}{4}$

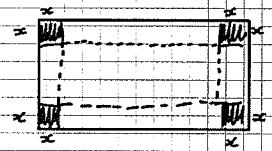
$x = 11.4cm$ too big ∴ reject

$x = 1.10cm$ too small ∴ reject

∴ $\boxed{\text{Square is } 5cm \times 5cm \quad \text{Dimensions } 10cm \times 5cm \times 5cm}$

$20-2(x) \quad 15-2(x) \quad x$

(2) Box: 40cm x 30cm piece of cardboard. Dimensions if volume is 3000cm³?



$(30-2x)(40-2x)(x) = 3000$

$(1200 - 140x + 4x^2)(x) = 3000$

$4x^3 - 140x^2 + 1200x - 3000 = 0 \quad \div 4$

$x^3 - 35x^2 + 300x - 750 = 0$

* $x > 0$

$x < 15$

Options: 1, 2, 3, 5, 6, 10

$$\begin{array}{r|rrrr} 5 & 1 & -35 & 300 & -750 \\ & \downarrow & 5 & -150 & 750 \\ \hline & 1 & -30 & 150 & 0 \end{array}$$

$x^2 - 30x + 150$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{30 \pm \sqrt{30^2 - 4(1)(150)}}{2} = \frac{30 \pm \sqrt{300}}{2}$$

$x = 23.66 \,/\, 6.34$ ✓

too big

* < 15

5cm x 20cm x 30cm

6.34cm x 17.32cm x 27.32cm

x $30-2x$ $40-2x$

pg 102 #1, 5

+ Review: pg 104

#1-50

* Not #13, 16, 30, 36

Note Title

Ch 2 Review

9/27/2012

① Given the graph of $f(x)=ax^3+bx^2+cx+d$ what must be true about a and d?

A) $a>0, d>0$

B) $a>0, d<0$

C) $a<0, d>0$

D) $a<0, d<0$

B

② What is the remainder when $x^{32}-1$ is divided by $x+1$?

$(-1)^{32}-1$

$1-1$

$\boxed{0}$

③ When x^3-x^2+kx+5 is divided by $x+2$, the remainder is 1. Find the value of k.

$(-2)^3-(-2)^2+k(-2)+5=1$

$-8-4-2k+5=1$

$-2k=8$

$\boxed{k=-4}$

OR

$$\begin{array}{r|rrrr} -2 & 1 & -1 & k & 5 \\ & \downarrow & -2 & 6 & -2k-12 \\ \hline & 1 & -3 & k+6 & \boxed{-2k-7} \end{array}$$

$-2k-7=1$

$-2k=8$

$\boxed{k=-4}$

④ $(x^4+3x^3+5x^2+21x-13)\div(x^2+3x-2)$ using long division.

$$\begin{array}{r} x^2 \quad + \quad 7 \\ x^2+3x-2 \,\overline{)\, x^4+3x^3+5x^2+21x-13} \\ -(x^4+3x^3-2x^2) \downarrow \quad \downarrow \\ 7x^2+21x-13 \\ -(7x^2+21x-14) \\ \hline 1 \end{array}$$

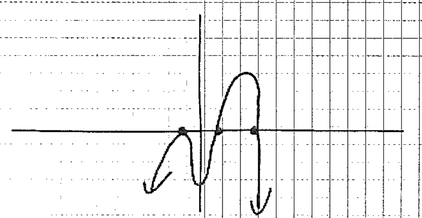
$$\frac{x^4+3x^3+5x^2+21x-13}{x^2+3x-2}=x^2+7+\frac{1}{x^2+3x-2}$$

OR

$$x^4+3x^3+5x^2+21x-13=(x^2+7)(x^2+3x-2)+1$$

(5) Given $f(x) = -(x+1)^2(x-1)(x-3)$, for what values of x is

a) $f(x) < 0$?



$\boxed{x < -1,\ -1 < x < 1,\ x > 3}$

b) $f(x) \geq 0$

$\boxed{x = -1,\ 1 \leq x \leq 3}$

(6) Given $5x^4 + 12x^3 - 101x^2 + 48x + 36$

a) Factor and b) Solve

Options: $\pm 1, \pm 2, \pm 3 \ldots$

$$\begin{array}{r|rrrrr} 1 & 5 & 12 & -101 & 48 & 36 \\ \downarrow & & 5 & 17 & -84 & -36 \\ \hline & 5 & 17 & -84 & -36 & 0 \end{array}$$

$5x^3 + 17x^2 - 84x - 36$

$$\begin{array}{r|rrrr} 3 & 5 & 17 & -84 & -36 \\ \downarrow & & 15 & 96 & 36 \\ \hline & 5 & 32 & 12 & 0 \end{array}$$

$5x^2 + 32x + 12$

$5x^2 + 30x + 2x + 12$

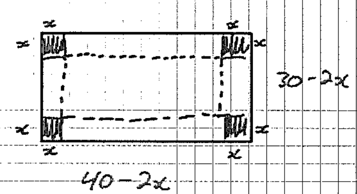
$5x(x+6) + 2(x+6)$

$(5x+2)(x+6)$

a) $(x-1)(x-3)(5x+2)(x+6)$

b) $x = 1, 3, -\frac{2}{5}, -6$

(7) Box: 40cm x 30cm piece of cardboard. Dimensions if volume is 3000cm³?



$(30-2x)(40-2x)(x) = 3000$

$(1200 - 140x + 4x^2)(x) = 3000$

$4x^3 - 140x^2 + 1200x - 3000 = 0$

$x^3 - 35x^2 + 300x - 750 = 0$

$$\begin{array}{r|rrrr} 5 & 1 & -35 & 300 & -750 \\ \downarrow & & 5 & -150 & 750 \\ \hline & 1 & -30 & 150 & 0 \end{array}$$

$x^2 - 30x + 150$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{30 \pm \sqrt{30^2 - 4(1)(150)}}{2} = \frac{30 \pm \sqrt{300}}{2}$

$x = 23.66 / 6.34$ ✓

too big

$* < 15$

$\boxed{\begin{array}{l} 5cm \times 20cm \times 30cm \\ 6.34cm \times 17.32cm \times 27.32cm \end{array}}$

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#1-50

* Not #13, 16, 30, 36

$x \quad 30-2x \quad 40-2x$