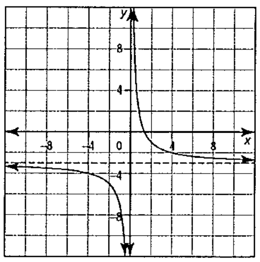
Chapter 3 Review2

Section 3.1

1. Match each function with its graph.

a) $y=\frac{4}{x}+3$ **b)** $y=\frac{4}{x+3}$ **c)** $y=\frac{4}{x-3}$ **d)** $y=\frac{4}{x}-3$

A



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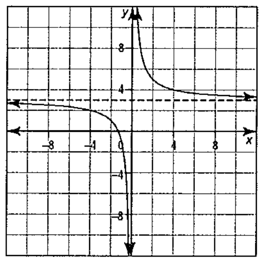

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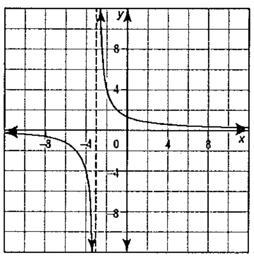

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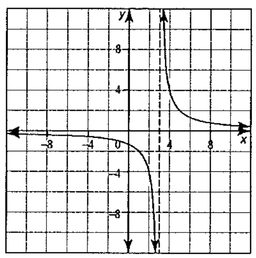


2. Graph $y=\frac{5}{x-2}$. Determine the following characteristics:

- non-permissible value(s)
- behaviour near NPV's
- end behaviour
- domain and range
- equation of vertical and horizontal asymptotes

3. Sketch and graph each function. Identify the domain and range, intercepts, and asymptotes.

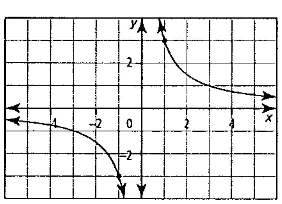
a) $y=\frac{3}{x-1}$ **b)** $y=\frac{2}{x}+6$ **c)** $y=\frac{5}{x+4}-2$ **d)** $y=\frac{1}{x+2}+8$

4. Graph and identify any asymptotes and intercepts.

a) $y=\frac{2x+5}{x-1}$ **b)** $y=\frac{4x-3}{x+2}$

5. Write the equation of each in the form $y=\frac{a}{x-h}+k$.

a)



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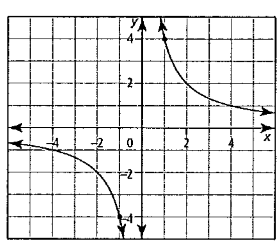

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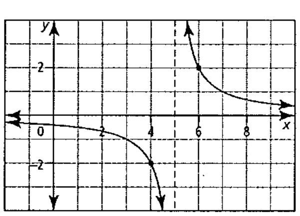
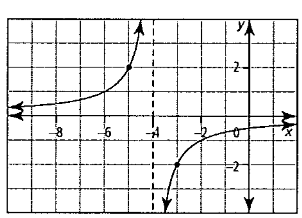


d)



6. The rational function $y=\frac{a}{x-5}+k$ passes through points (6, 7) and (4, 1).

a) Determine the value of *a* and *k*.
b) Graph the function.

7. Sketch $y=\frac{1}{x^2}$ and $y=\frac{1}{x^2+6x+9}$ on the same set of axes. Describe how one is a transformation of the other.

8. Use a table of values and a graph to analyse the function $y=\frac{2x-1}{x-7}$. Then, complete the table.

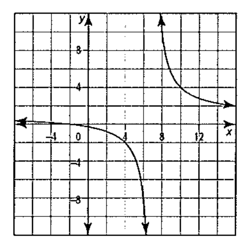
Characteristic	$y=\frac{2x-1}{x-7}$
Non-permissible value	
Behaviour near non-permissible value	
End behaviour	
Domain and Range	
Equation of vertical / horizontal asymptotes	

9. The distance between two cities is 351 km.

a) Write an expression to calculate the time, *t*, in hours, it takes to travel distance, *d*, in km, at a speed of *s* km/h.
b) How long it will take to travel at a speed of 65 km/h?
c) If the trip took 5 h, determine the speed, *s*, in km/h.

Section 3.2

1. Explain the behaviour at each non-permissible value in the graph of the rational function $y=\frac{x^2+5x+6}{x^2-4x-21}$.



2. Explain whether $y=\frac{x+2}{x^2+3x+2}$ has an asymptote or a point of discontinuity.

3. Complete the table for the given rational function.

Characteristic	$y=\frac{(x+3)(x-2)}{(x+5)(x+3)}$
Non-permissible value(s)	
Feature exhibited at each non-permissible value	
Behaviour near each non-permissible value	
Domain and Range	

4. Create a table of values for each function for values near its non-permissible value(s). Determine whether each graph has a point of discontinuity or an asymptote.

a) $y=\frac{x^2+5x+4}{x+1}$ b) $y=\frac{x^2+5x-14}{x^2-6x+8}$

5. Determine the vertical asymptotes, points of discontinuity, and intercepts. Graph.

a) $y=\frac{x^2+5x}{x^2+7x+10}$ b) $y=\frac{x^2-7x+12}{x^2-9}$

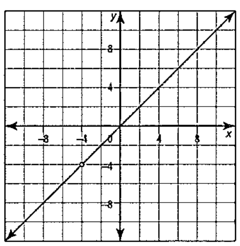
c) $y=\frac{x^2+5x+4}{x+1}$ d) $y=\frac{2x^2+5x-3}{x+3}$

6. Complete the table and compare the behaviour of the two functions near any non-permissible values.

Characteristic	$y=\frac{x^2-3x}{3x-9}$	$y=\frac{x^2+3x}{3x-9}$
Non-permissible value(s)		
Feature exhibited at each NPV		
Behaviour near each NPV		

7. Match the equation of each rational function with the most appropriate graph. Explain your reasoning.

a) $y=\frac{x+4}{x^2-3x-4}$ b) $y=\frac{x+4}{x^2+5x+4}$ c) $y=\frac{x^2+4x}{x+4}$

A)


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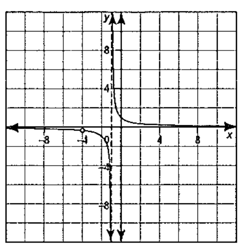

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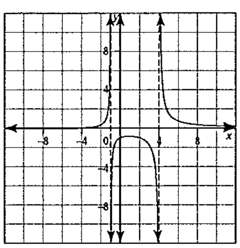
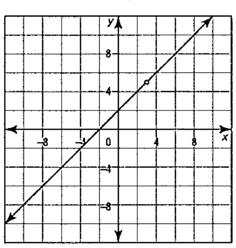


8. Write the equation for each graphed rational function.

a)


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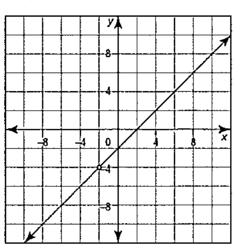
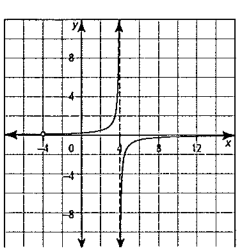
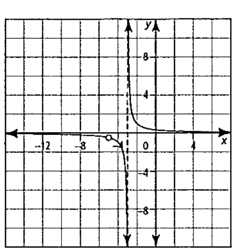


c)


d)


9. Write the equation of a rational function that has an asymptote at $x=2$, a point of discontinuity at $x=-2.5$, and passes through (6, –3).

Section 3.3

1. Solve each equation algebraically.

a) $\frac{2}{x-1}-5=\frac{4}{x-1}$ b) $\frac{3}{x+5}+\frac{1}{2}=\frac{x+3}{x+5}$

c) $\frac{8}{x}+\frac{x+6}{3x}+\frac{x-4}{6x}=\frac{8}{9}$ d) $\frac{x^2+2}{x}=\frac{2x+1}{2}$

2. Solve algebraically. Check your solutions.

a) $x=\frac{13}{x-9}-3$ b) $x=\frac{x+5}{x-3}+4$

c) $x+4=\frac{4x+2}{x-7}$ d) $x+3=\frac{x^2}{2-x}$

3. a) Determine the roots of $\frac{5}{x}+x-6=0$ algebraically.

b) Graph $y=\frac{5}{x}+x-6$ and determine the *x*-intercepts.

c) Explain the connection between the roots of the equation and the *x*-ints of the graph of the function.

4. Solve each of the following equations by graphing each side of the equation as a separate function.

a) $3x=\frac{6x}{2x-5}$ b) $\frac{17-3x+x^2}{x-1}=2x-5$

c) $\frac{2x^2-16x}{2x-1}=3x-2$

5. Rearrange as a single function and then graph.

a) $\frac{x}{x-3}+4=x$ b) $\frac{4}{x+1}=\frac{2}{x-1}$

6. Solve algebraically. Round to 2 decimal places.

a) $x-1=\frac{x}{x-4}$ b) $x+3=\frac{x+2}{x-1}$ c) $\frac{3}{5x-2}+x=5$

7. Determine the approximate solution(s) to each rational equation graphically, to the nearest hundredth.

a) $\frac{2x}{x-1}+3x=\frac{x-3}{x+1}$ b) $4-\frac{3}{x-7}=9-\frac{x+3}{x}$

8. Solve the equation $\frac{18}{n^2-9}+1=\frac{n}{n+3}$ algebraically.

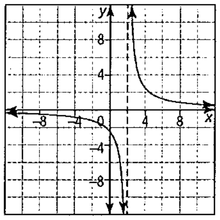
9. It takes James 9 h longer to construct a fence than it takes Carmen. If they work together, they can construct the fence in 20 h. How long would it take each of them, working alone, to construct the fence?

Answers Section 3.1

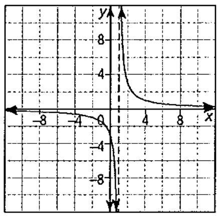
Characteristic	$y=\frac{5}{x-2}$

1 a) B **b)** C **c)** D **d)** A

2.

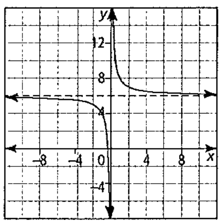


3a)

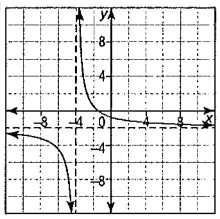


D: $\{x \mid x \neq 1, x \in R\}$; R: $\{y \mid y \neq 0, y \in R\}$;
y-int: (0, –3); **asymptotes**: $x = 1$, $y = 0$

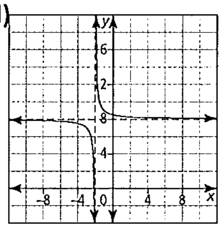
b)



c)



d)

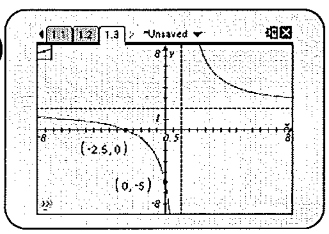


b) D: $x \neq 0$ **R:** $y \neq 6$
Int.: $\left(-\frac{1}{3}, 0\right)$
asym.: $x = 0$, $y = 6$

c) D: $x \neq -4$ **R:** $y \neq -2$
Int: (0, –0.75), (–1.5, 0)
asym: $x = -4$, $y = -2$

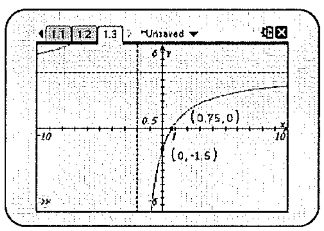
d) D: $x \neq -2$ **R:** $y \neq 8$
Int: (0,8.5), (–2.125,0)
asym: $x = -2$, $y = 8$

4a)



asymptotes: $x = 1$, $y = 2$;
intercepts: (–2.5, 0), (0, –5)

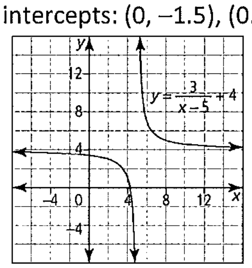
b)



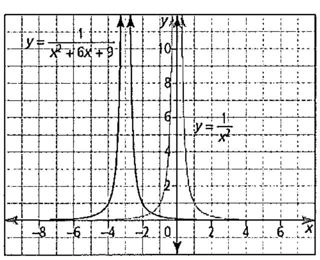
asymptotes: $x = -2$, $y = 4$;
intercepts: (0, –1.5), (0.75, 0)

5a) $y = \frac{3}{x}$ **b)** $y = \frac{4}{x}$ **c)** $y = \frac{2}{x-5}$

d) $y = -\frac{2}{x+4}$ **6. a)** $a = 3$, $k = 4$ **6b)**



7.



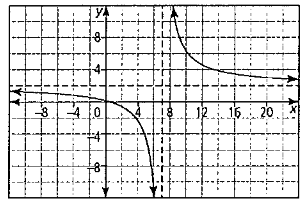
$y = \frac{1}{x^2 + 6x + 9}$ is translated 3 units left.

8.

x	y	x	y
–5	0.92	10	6.33
–2	0.56	13	4.17
1	0.17	16	3.44

8.

Characteristic	$y = \frac{2x-1}{x-7}$
NPV	$x = 7$
Behaviour near	As x approaches 7, $\lvert y \rvert$ becomes very larg



9. a) $t = \frac{d}{s}$ **b)** $t = \frac{351}{65} = 5.4$, so 5.4 hours or 5 h and 24 min
c) 70.2 km/h

Section 3.2

1. point of discontinuity at $(-3, \frac{1}{10})$ vertical asymptote: $x = 7$

2. Factor the denominator: $y = \frac{x+2}{(x+2)(x+1)}$. Since $(x + 2)$ appears in the numerator and denominator, the graph will have a point of discontinuity at (–2, –1). The factor $(x + 1)$ appears in the denominator only, so there will be an asymptote at $x = -1$.

3.

Characteristic	$y = \frac{(x+3)(x-2)}{(x+5)(x+3)}$
NPV's	$x = -5$ and $x = -3$
Feature exhibited at each NPV	asymptote at $x = -5$; point of discontinuity at (–3, –2.5)
Behaviour near each NPV	As x approaches –5, $\lvert y \rvert$ becomes very large. As x approaches –3, y approaches –2.5.
Domain	$\{x \mid x \neq -3, -5, x \in R\}$
Range	$\{y \mid y \neq 1, -\frac{5}{2}, y \in R\}$

4a)

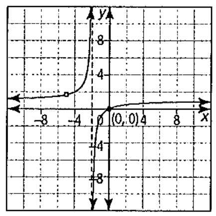
x	y	x	y
–0.9	3.1	–1.0001	2.9999
–0.99	3.01	–1.001	2.999
–0.999	3.001	–1.01	2.99
–0.9999	3.0001	–1.1	2.9
–1	undefined		

As x approaches –1, y approaches 3.

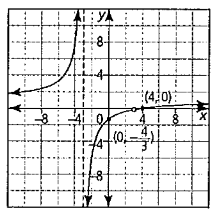
b) As x approaches 2, y approaches –4.5, and as x approaches 4, $\lvert y \rvert$ becomes very large, approaching negative infinity or positive infinity.

x	y	x	y
1.9	–4.238 095 24	2.0001	–4.500 275 01
1.99	–4.472 636 82	2.001	–4.502 751 38
1.999	–4.497 251 37	2.01	–4.527 638 19
1.9999	–4.499 725 01	2.1	–4.789 473 68
2	undefined		

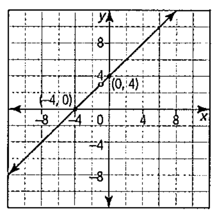
5a) vertical asymptote: $x = -2$; point of discontinuity at $(-5, \frac{5}{3})$; x-intercept: (0, 0); y-intercept: (0, 0)



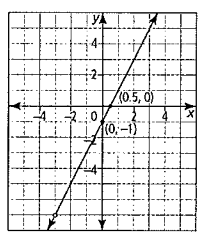
b) vertical asymptote: $x = -3$; point of discontinuity at $(3, -\frac{1}{16})$; x-intercept: (4, 0); y-intercept: $(0, -\frac{4}{3})$



c) no vertical asymptote; point of discontinuity at (–1, 3); x-intercept: (–4, 0); y-intercept: (0, 4)



d) no vertical asymptote; point of discontinuity at (–3, –7); x-intercept: (0.5, 0); y-intercept: (0, –1)



6)

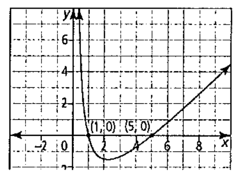
Characteristic	$y = \dfrac{x^2 - 3x}{3x - 9}$	$y = \dfrac{x^2 + 3x}{3x - 9}$
NPV	$x = 3$	$x = 3$
Feature exhibited at each NPV	point of discontinuity	asymptote
Behaviour near each NPV	As x approaches 3, y approaches 1.	As x approaches 3, $\lvert y \rvert$ becomes very large.

7. a) C; When factored, the function has two NPVs in the den., which aren't in the num.. Therefore, graph with two asymptotes.
b) B; In factored form, the function has one NPV that appears in both the numerator and denominator, and another NPV that is only in the denominator. Therefore, graph with one asymptote and one point of discontinuity
c) A; Example: In factored form, one non-permissible value appears in the numerator and denominator. Therefore, the graph has a point of discontinuity, but no asymptote.

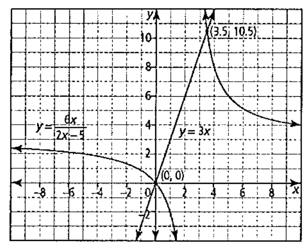
8. a) $y = \dfrac{(x-3)(x+2)}{(x-3)}$ **b)** $y = \dfrac{(x-2)(x+2)}{(x+2)}$ **c)** $y = \dfrac{(x+4)}{(4-x)(4+x)}$
d) $y = \dfrac{(x+5)}{(x+3)(x+5)}$ **9.** Example: $y = \dfrac{-12(2x+5)}{(x-2)(2x+5)}$

Section 3.3

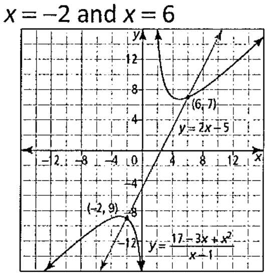
1. a) $x = \frac{3}{5}$ **b)** $x = 5$ **c)** $x = 24$ **d)** $x = 4$ **2. a)** $x = 10$ and $x = -4$
2b) $x = 7$ and $x = 1$ **c)** $x = 10$ and $x = -3$ **d)** $x = \frac{3}{2}$ and $x = -2$
3. a) $x = 5$ and $x = 1$
b)



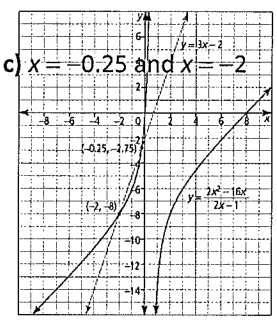
c) The value of the function is 0 when the value of x is 1 or 5. The x-intercepts of the graph of the function are the same as the roots of the corresponding equation.



b) $x = -2$ and $x = 6$

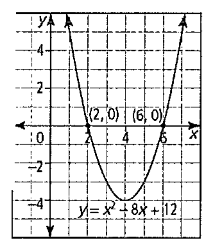


c) $x = -0.25$ and $x = -2$

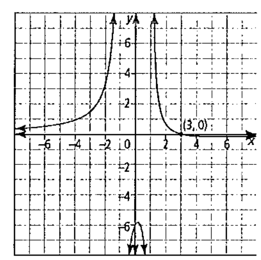


5. a) $0 = x^2 - 8x + 12$ **b)** $y = \dfrac{6 - 2x}{x^2 - 1}$

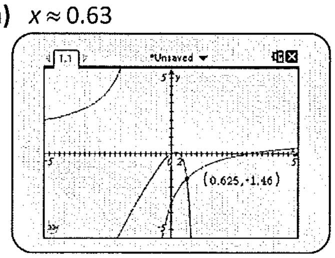
$x = 2$ and $x = 6$

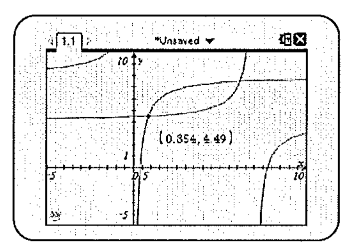


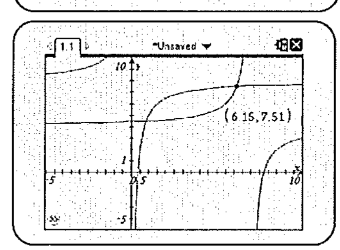
$x = 3$



6. a) $x \sim 0.76$, ~ 5.24 **b)** $x \sim -2.79$, ~ 1.79 **c)** $x \sim 0.53$, ~ 4.87
7a) $x \approx 0.63$ **b)** $x \approx 0.85$ and $x \approx 6.15$







8. The solution $n = 3$ is a non-permissible value, so there is no solution.
9. Carmen: 36 h; James: 45 h