Note Title 2012-10-01

4.3 RATIONAL FUNCTIONS

A rational function is a function whose equation can be written in the form: $f(x) = \frac{m(x)}{n(x)}$ where $m(x)$ & $n(x)$ are polynomial functions & $n(x) \neq 0$.

An **ASYMPTOTE** of a graph is a vertical or horizontal line that a part of the graph gets very close to but never reaches.

VERTICAL ASYMPTOTES

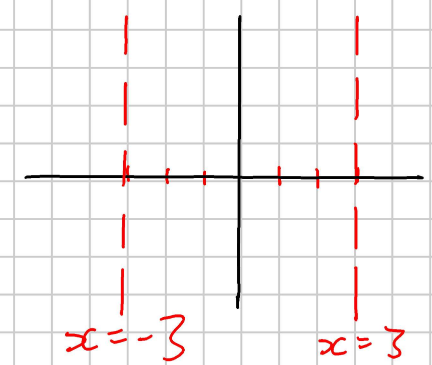
- can be determined by setting the denominator equal to zero and solving for x.

<u>Ex</u> $f(x) = \dfrac{x}{x^2-9}$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3 \rightarrow$ VERTICAL ASYMPTOTES



(1) Determine the vertical asymptotes.

a) $y = \dfrac{x^2}{x-1}$

$x - 1 = 0$

$\boxed{x = 1}$

b) $y = \dfrac{x+7}{x^2+9x+20}$

$x^2 + 9x + 20 = 0$

$(x+4)(x+5) = 0$

$\boxed{x = -4,\ x = -5}$

HORIZONTAL ASYMPTOTES

- can be found by looking at the "end behaviour"

as $x \to \infty$ or as $x \to -\infty$

Given: $f(x) = \dfrac{ax^m + \ldots + cx + d}{bx^n + \ldots + ex + f}$

1. If $m < n$ $\to$ $y = 0$ (x-axis) is a HORIZ. ASYMPTOTE
2. If $m = n$ $\to$ $y = \frac{a}{b}$ (ratio of leading coefficients) is a HORIZ. ASYMPTOTE
3. If $m > n$ $\to$ NO HORIZ. ASYMPTOTE

(2) Determine the horizontal asymptotes.

a) $y = \dfrac{2x}{3x-1}$

$m = n = 1$

$y = \dfrac{a}{b}$

$\boxed{y = \dfrac{2}{3}}$

b) $f(x) = \dfrac{1-2x}{x^2-4x+3}$

$m = 1,\ n = 2$

$m < n$

$\boxed{y = 0}$

c) $g(x) = \dfrac{2x^2-3x+1}{x+4}$

$m = 2,\ n = 1$

$m > n$

$\boxed{\text{NO HORIZONTAL ASYMPTOTE}}$

HOLES IN RATIONAL FUNCTIONS

Sometimes a rational function simplifies to a different function and eliminates one or more of the vertical asymptotes.

<u>Ex</u> $f(x) = \dfrac{x+2}{x^2+x-2}$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2 \quad x=1$

Appears to have 2 vertical asymptotes however...

$$y = \frac{\cancel{x+2}}{\cancel{(x+2)}(x-1)} \qquad y = \frac{1}{x-1}$$

→ only 1 vertical asymptote $x = 1$

→ Hole at $x = -2$

* "POINT OF DISCONTINUITY"

$$\left(-2, -\frac{1}{3}\right)$$

$$\uparrow$$

$$y = \frac{1}{-2-1}$$

3. Determine the asymptotes and holes of the rational functions:

a) $y = \dfrac{x-3}{x^2-x-6}$

$y = \dfrac{\cancel{x-3}}{\cancel{(x-3)}(x+2)}$

HOLE

$y = \dfrac{1}{x+2}$

$\boxed{\begin{array}{l} \text{Vert. Asymptote: } x = -2 \\ \text{Horiz. Asymptote: } y = 0 \\ \text{Hole: } \left(3, \tfrac{1}{5}\right) \end{array}}$

b) $y = \dfrac{x^2+7x+12}{x^2-x-20}$

$y = \dfrac{\cancel{(x+4)}(x+3)}{(x-5)\cancel{(x+4)}}$ $\qquad y = \dfrac{x+3}{x-5}$

$$\boxed{\begin{array}{ll} \text{Vert. Asym:} & x = 5 \\ \text{Horiz. Asym:} & y = 1 \\ \text{Hole:} & \left(-4, \frac{1}{9}\right) \end{array}}$$

$y = \dfrac{-4+3}{-4-5} = \dfrac{-1}{-9}$

c) $y = \dfrac{x^2 - 4}{x^3 - 4x}$

$y = \dfrac{\cancel{(x-2)}\cancel{(x+2)}}{x\cancel{(x-2)}\cancel{(x+2)}}$ OR $y = \dfrac{\cancel{x^2-4}}{x\cancel{(x^2-4)}}$ $y = \dfrac{1}{x}$

$y = \dfrac{1}{x}$

Vert. Asymptote: $x = 0$

Horiz. Asymptote: $y = 0$

Holes: $(2, \frac{1}{2})$ + $(-2, \frac{-1}{2})$

4. Determine the x and y-intercepts, asymptotes and holes for: $y = \frac{(x+6)(x+3)}{(x-2)^2}$

x-int: $0 = (x+6)(x+3)$

$$\boxed{x = -6, \ x = -3}$$

OR

$(-6,0)$, $(-3,0)$

y-int: $y = \frac{(0+6)(0+3)}{(0-2)^2}$

$y = \frac{(6)(3)}{4}$

$$\boxed{y = \frac{18}{4} = \frac{9}{2} = 4.5}$$

$(0, 4.5)$

Vert. Asym: $x = 2$

Horiz. Asym: $y = 1$

* Nothing cancels ∴ No holes

Pg 180

#2, 4 (a, c, e ...)