# 4.3 Rational Functions - Day 2

Determine the equations of any asymptotes, x/y intercepts, points of discontinuity ("holes") and the domain range.

(1) a) $y = \dfrac{x^2 + 2x}{x^2 - 4} \rightarrow$   $y = \dfrac{x(x+2)}{(x-2)(x+2)} \rightarrow y = \dfrac{x}{x-2}$   $y = \dfrac{-2}{-2-2} = \dfrac{-2}{-4} = \dfrac{1}{2}$

Vertical Asymptotes    Horizontal Asymptote    Holes

$\boxed{x = 2}$    $m = n$    $x = -2$

$\boxed{y = 1}$    $y = \dfrac{1}{1}$    $\dfrac{x+2}{x+2}$

$\boxed{\left(-2, \dfrac{1}{2}\right)}$

$y = 1$

**x-intercept**
$(y=0)$

$$0 = \frac{x}{x-2}$$

$$0 = x$$

$$x = 0$$

$$\boxed{x\text{-int } (0,0)}$$

**y-intercept**
$(x=0)$

$$y = \frac{0}{-2}$$

$$y = 0$$

$$\boxed{y\text{-int } (0,0)}$$

**Domain/Range**

$$\boxed{\begin{array}{l} D: x \neq \pm 2 \\ R: y \neq 1, \frac{1}{2} \end{array}}$$

b) $y = \dfrac{2x+4}{x^2+1}$

**Vertical Asymptotes**

$x^2 + 1 = 0$

$x^2 = -1$

$\emptyset$

No Vertical Asymptotes

**Horizontal Asymptote**

$m < n$

$y = 0$

**Holes**

$y = \dfrac{2x+4}{x^2+1}$

$y = \dfrac{2(x+2)}{x^2+1}$

No Holes

"Nothing Cancels"

x-intercept
$(y=0)$

$0 = \dfrac{2x+4}{x^2+1}$

$0 = 2x+4$

$x = -2$

$(-2, 0)$

y-intercept
$(x=0)$

$y = \dfrac{4}{1}$

$y = 4$

$(0, 4)$

Domain/Range

$D: x \in \mathbb{R}$

$R: y \neq 0$

② Write the equation of a rational function with the following:

a) 
- vertical asymptote of $x = -3$
- horizontal asymptote of $y = 4$
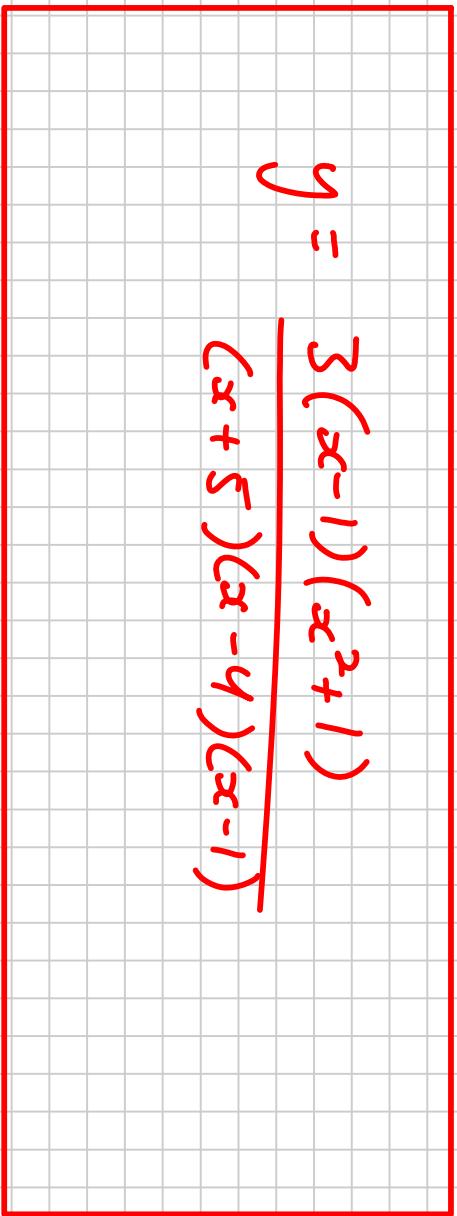- hole at $(2, 0)$

$$y = \frac{4(x-2)}{(x+3)(x-2)}$$

but degree needs to
be the same on
top & bottom

$$\boxed{y = \frac{4(x-2)^2}{(x+3)(x-2)}}$$

b) 
- vertical asymptotes at $x=-5$, $x=4$
- horizontal asymptote at $y=3$
- hole at $x=1$
- no $x$-intercepts

$$y = \frac{3(x-1)(x^2+1)}{(x+5)(x-4)(x-1)}$$

$$y = \frac{3(x-1)}{(x+5)(x-4)(x-1)}$$

Pg 180
#2, 4
$(b, d, f, \ldots)$