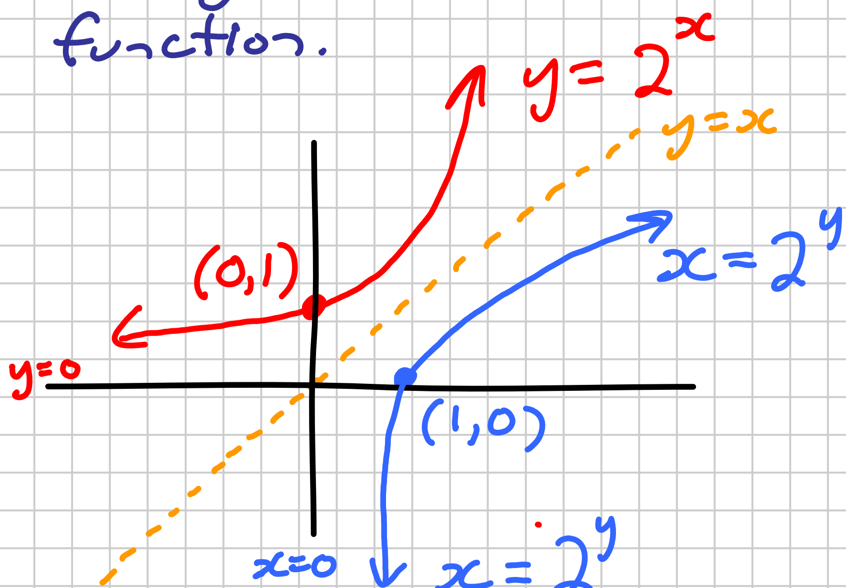
5.2 Logarithmic Functions Pt. 1

A logarithmic function is the inverse of an exponential function.



$x = 2^y$

or

$y = \log_2 x$

"log of x base 2"

$$\boxed{\begin{array}{l} \text{Given } y = \log_b x \\ x > 0 \\ b > 0,\ b \neq 1 \end{array}}$$

Logs: D: $x > 0$

R: $y \in \mathbb{R}$

Exponential Form | Logarithmic Form

$$x = b^y \longrightarrow y = \log_b x$$

① Express in logarithmic form

a) $n = 4^x$

$x = \log_4 n$

b) $a = b^c$

$c = \log_b a$

c) $2^{-4} = \frac{1}{16}$

$-4 = \log_2 \frac{1}{16}$

2. Express in exponential form

a) $\log_k t = x$

$$\boxed{k^x = t}$$

b) $\log 10000 = 4$

$$\square^4 = 10000$$

$$\boxed{10^4 = 10000}$$

$\log x = \log_{10} x$ *No base can assume 10

COMMON LOGARITHM

3 Evaluate

a) $\log_2 64 = x$

$$2^x = 64$$

$$\boxed{x = 6}$$

b) $\log_4 \frac{1}{16} = x$

$$4^x = \frac{1}{16}$$

$$4^x = 4^{-2}$$

$$\boxed{x = -2}$$

c) $\log_4 4^5$

$4^x = 4^5$

$\boxed{x = 5}$

d) $\log_{1000} 1000^7$

$\boxed{7}$

<u>* NOTE:</u> $4^{\log_4 x} = x$

$5^{\log_5 4} = 4$

e) $\log_2 7$

Change of Base Rule

$$= \frac{\log 7}{\log 2}$$

$$= \boxed{2.81}$$

f) $\log_7 2$

$$\frac{\log 2}{\log 7} = \boxed{0.36}$$

4 Solve

a) $\log_x 125 = 3$

$$x^3 = 125$$

$$\boxed{x = 5}$$

b) $3 = \log_2\left(\frac{x}{4}\right)$

$$2^3 = \frac{x}{4}$$

$$8 = \frac{x}{4}$$

$$\boxed{x = 32}$$

c) $\log_{\sqrt{2}} 16 = x$

$\sqrt{2}^{x} = 16$

"square both sides"

$2^{\frac{x}{2}} = 2^{4}$ OR $2^{x} = (2^{4})^{2}$

$\frac{x}{2} = 4$

$2^{x} = 2^{8}$

$\boxed{x = 8}$

$\boxed{x = 8}$

d) $\log_{-3x} 36 = 2$

$(-3x)^{2} = 36$

$9x^{2} = 36$

$x^{2} = 4$

$x = \pm 2$

$* \boxed{b > 0}$

$\therefore \boxed{x = -2}$

e) $\log_x 32 = \frac{5}{2}$

$$\left(x^{\cancel{\frac{5}{2}}}\right)^{\cancel{\frac{2}{5}}} = (32)^{\frac{2}{5}}$$

$$x = \left(\sqrt[5]{32}\right)^2$$

$$x = (2)^2$$

$$\boxed{x = 4}$$

f) $x = \log_5 0.04$

$$5^x = 0.04$$

$$5^x = \frac{4}{100}$$

$$5^x = \frac{1}{25}$$

$$5^x = 5^{-2}$$

$$\boxed{x = -2}$$

Pg 211 #1-4(a,c,e...)