5.4 Exponential and Logarithmic Equations Pt. 1

Note Title 2012-10-19

SOLVE

① $3^{x+2} = 5^{2x-3}$

$\log 3^{x+2} = \log 5^{2x-3}$

$(x+2)\log 3 = (2x-3)\log 5$

$x\log 3 + 2\log 3 = 2x\log 5 - 3\log 5$

$x\log 3 - 2x\log 5 = -3\log 5 - 2\log 3$

$x(\log 3 - 2\log 5) = -3\log 5 - 2\log 3$

$$x = \frac{-3\log 5 - 2\log 3}{\log 3 - 2\log 5}$$

$$x = \frac{(-3\log 5 - 2\log 3)}{(\log 3 - 2\log 5)} = 3.31$$

② $7 \cdot 2^x = 5^{x-2}$

$\log(7 \cdot 2^x) = \log 5^{x-2}$

$\log 7 + \log 2^x = \log 5^{x-2}$

$\log 7 + x \log 2 = (x-2)\log 5$

$\log 7 + x \log 2 = x \log 5 - 2 \log 5$

$x \log 2 - x \log 5 = -2 \log 5 - \log 7$

$x(\log 2 - \log 5) = -2 \log 5 - \log 7$

$x = \dfrac{(-2 \log 5 - \log 7)}{(\log 2 - \log 5)} = \underline{5.64}$

(3) $4(7^{x+2}) = 8^{2x-3}$

$\log 4 + \log 7^{x+2} = \log 8^{2x-3}$

$\log 4 + (x+2)\log 7 = (2x-3)\log 8$

$\log 4 + x\log 7 + 2\log 7 = 2x\log 8 - 3\log 8$

$x\log 7 - 2x\log 8 = -3\log 8 - \log 4 - 2\log 7$

$x(\log 7 - 2\log 8) = -3\log 8 - \log 4 - 2\log 7$

$$x = -\frac{3\log 8 - \log 4 - 2\log 7}{\log 7 - 2\log 8} = \underline{5.20}$$

④ $\log_3 3y - \log_3 4 = \log_3 6$

$\log_3\left(\frac{3y}{4}\right) = \log_3 6$ OR $\log_3 3y = \log_3 6 + \log_3 4$

$\frac{3y}{4} = 6$

$3y = 24$

$\boxed{y = 8}$

$\log_3 3y = \log_3 24$

$3y = 24$

$\boxed{y = 8}$

(5) $2\log(4-x) - \log 3 = \log(10-x)$

$\log(4-x)^2 = \log(10-x) + \log 3$

$\log(4-x)^2 = \log(10-x)(3)$

$(4-x)(4-x) = 30-3x$

$16-8x+x^2 = 30-3x$

$x^2-5x-14 = 0$

$(x-7)(x+2) = 0$ $\quad x \neq 7,$ $\boxed{x = -2}$

(6) $\log_3(3x-1) - \log_3(x-1) = 4$

$\log_3\left(\frac{3x-1}{x-1}\right) = 4$

$3^4 = \frac{3x-1}{x-1}$

$\frac{81}{1} = \frac{3x-1}{x-1}$

$81x - 81 = 3x - 1$

$78x = 80$

$x = \frac{40}{39}$

(7) $\log_4(x-5) + \log_4(x-2) = 1$

$\log_4(x-5)(x-2) = 1$

$4^1 = (x-5)(x-2)$

$4 = x^2 - 7x + 10$

$0 = x^2 - 7x + 6$

$(x-6)(x-1) = 0$

$\boxed{x = 6}$, ~~$x \neq 1$~~

(8) $2\log_3 x = \log_3 32 + \log_9 4$

$\log_3 x^2 = \log_3 32 + \log_3 2$

$\log_3 x^2 = \log_3 64$

$x^2 = 64$

$x = 8, -8$

$\boxed{x = 8}$

Pg 227

#1, 4