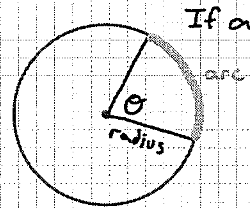
Note Title 2014-04-22

5.1 TRIG FUNCTIONS

One **RADIAN** is the measure of the angle formed by rotating the radius of a circle through an arc equal in length to the radius.

If arc = radius $\rightarrow \theta = 1$ radian



CONVERSIONS

Radians $\rightarrow$ Degrees multiply by $\dfrac{180^\circ}{\pi}$

Degrees $\rightarrow$ Radians multiply by $\dfrac{\pi}{180^\circ}$

① Express in radians (express answers in terms of π)

a) 75° b) 180° c) 90° d) 360°

$75^\circ\left(\dfrac{\pi}{180^\circ}\right)$ $180^\circ\left(\dfrac{\pi}{180^\circ}\right)$ $90^\circ\left(\dfrac{\pi}{180^\circ}\right)$ $360^\circ\left(\dfrac{\pi}{180^\circ}\right)$

$\boxed{\dfrac{5\pi}{12}}$ $\boxed{\pi}$ $\boxed{\dfrac{\pi}{2}}$ $\boxed{2\pi}$

② Express in degrees

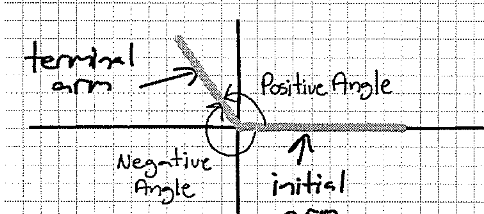
a) $\dfrac{4\pi}{3}$ b) $\dfrac{5\pi}{6}$ c) 2.5

$\dfrac{4(180^\circ)}{3}$ OR $\dfrac{4\cancel{\pi}}{3}\left(\dfrac{180}{\cancel{\pi}}\right)$ $\dfrac{5(180^\circ)}{6}$ $2.5\left(\dfrac{180^\circ}{\pi}\right)$

$\boxed{240^\circ}$ $\boxed{150^\circ}$ $\boxed{143.24^\circ}$

An angle is in **STANDARD POSITION** when:

- its vertex is at the origin $(0,0)$
- its initial arm is on the positive x-axis



Positive Angle: counterclockwise

Negative Angle: clockwise

CO-TERMINAL ANGLES

- angles with the same terminal arm
- to find coterminal angles $\pm n\,360°$ OR $\pm n\,2\pi$ (multiples of $360°$ or 2π) $n \in I$ ← n is an integer

③ List 2 angles coterminal with:

a) $70°$

$\boxed{430°,\ 790° \ldots \quad -290° \ldots}$

b) $\frac{\pi}{3}$

$\frac{2\pi \times 3}{1 \times 3} = \frac{6\pi}{3}$

$\frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}, \frac{13\pi}{3} \ldots$

$\frac{\pi}{3} - \frac{6\pi}{3} = -\frac{5\pi}{3} \ldots$

c) $\frac{2\pi}{5} \pm \frac{10\pi}{5}$

$\boxed{\frac{12\pi}{5}, \frac{22\pi}{5}, -\frac{8\pi}{5} \ldots}$

PRINCIPAL ANGLE - smallest positive coterminal angle

$* \quad 0° \leq \theta < 360° \;/\; 0 \leq \theta < 2\pi$

Ex Given $-30°$, the principal angle is $330°$ $(+360°)$

Ex

$70° \rightarrow$ P.A. $= 70°$ $\qquad 750° \rightarrow$ PA $= 30°$

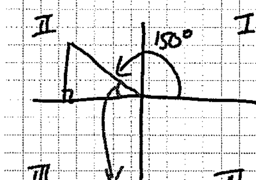
$400° \rightarrow$ P.A. $= 40°$

REFERENCE ANGLE - the positive, acute angle formed between the terminal arm and the x-axis.

$* \quad 0° \leq \theta \leq 90°$ OR $0 \leq \theta \leq \frac{\pi}{2}$

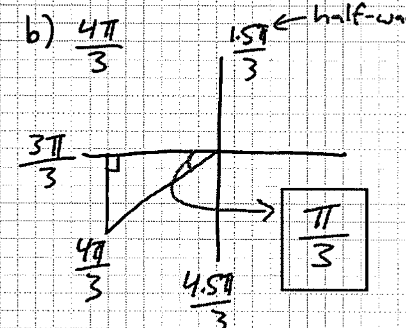
④ Determine the reference angle

a) $150°$



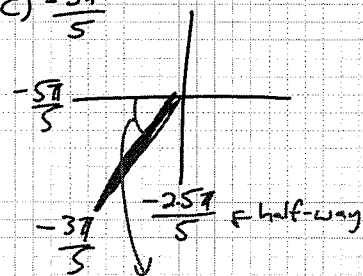
$\boxed{30°}$

b) $\frac{4\pi}{3}$



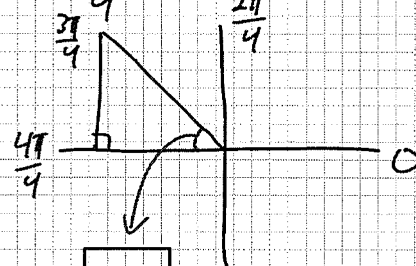
$\boxed{\frac{\pi}{3}}$

c) $\frac{-3\pi}{5}$



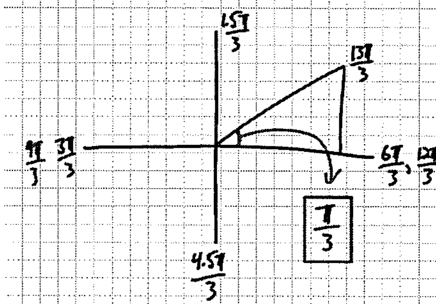
$\boxed{\frac{2\pi}{5}}$

d) $\frac{3\pi}{4}$



$\boxed{\frac{\pi}{4}}$

e) $\frac{13\pi}{3}$



$\boxed{\frac{\pi}{3}}$

OR

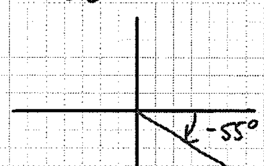
Determine co-terminal angle

$\frac{\pi}{3}$

$$\frac{13\pi}{3} - \frac{6\pi}{3} - \frac{6\pi}{3} = \frac{\pi}{3}$$

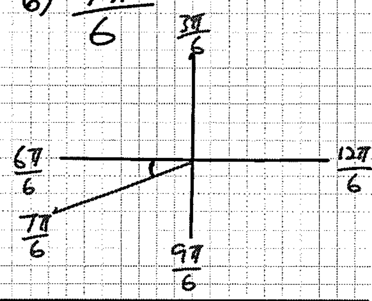
(5) Determine the quadrant of the terminal arm, the principal angle and the reference angle.

a) $-55°$



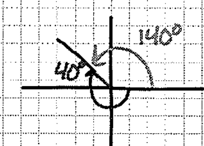
$\boxed{\text{IV}, 305°, 55°}$

b) $\frac{7\pi}{6}$



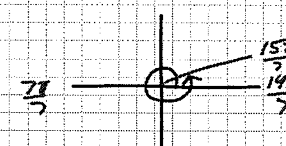
$\boxed{\text{III}, \text{P.A.} = \frac{7\pi}{6}, \text{RA} = \frac{\pi}{6}}$

c) $-220°$



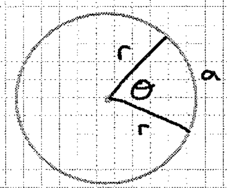
$\boxed{\text{II}, \text{P.A.} = 140°, \text{R.A.} = 40°}$

d) $\frac{15\pi}{7}$



$\boxed{\text{I}, \text{P.A.} = \frac{\pi}{7}, \text{R.A.} = \frac{\pi}{7}}$

Determining Arc Length



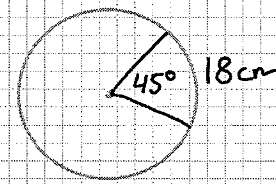
$a = r\theta$

a = arc length

r = radius

θ = sector angle in RADIANS

⑤ Determine the radius



$a = r\theta$

$r = \dfrac{a}{\theta}$ * θ must be in RADIANS

$(45°)\left(\frac{\pi}{180°}\right) = \frac{\pi}{4}$ $r = \dfrac{18}{\frac{\pi}{4}} = \boxed{22.92\text{cm}}$

Pg 213

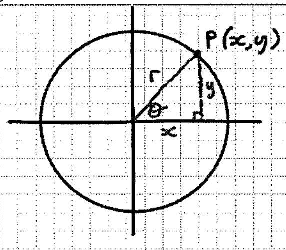
#1-13 (a, c, e)

② a) $\frac{1}{8}$ rotation (degrees) $= \frac{1}{8}(360°) = \boxed{45°}$

③ a) $\frac{1}{6}$ rotation (radians) $= \frac{1}{6}(2\pi) = \boxed{\frac{\pi}{3}}$

5.2 Trig Functions of Acute Angles

Note Title 2014-04-24



PRIMARY TRIG RATIOS

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

RECIPROCAL TRIG RATIOS

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

↑ cosecant ↑ secant ↑ cotan

$x^2+y^2=r^2$

* r is ALWAYS POSITIVE

① The point $(-5,-12)$ lies on the terminal arm of an angle θ in standard position.

Determine the exact values of the primary and reciprocal trig ratios.

$x=-5$
$y=-12$

$x^2+y^2=r^2$

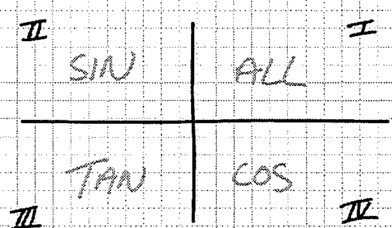
$25+144=r^2$

$r=\sqrt{169}$

$r=13$

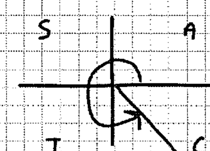
$$\boxed{\begin{array}{lll} \sin\theta = -\frac{12}{13} & \cos\theta = \frac{-5}{13} & \tan\theta = \frac{12}{5} \\ \csc\theta = \frac{13}{-12} & \sec\theta = \frac{13}{-5} & \cot\theta = \frac{5}{12} \end{array}}$$

In what quadrants are the trig ratios positive?

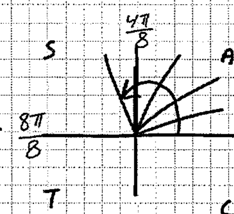


② State whether positive or negative

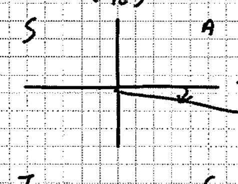
a) $\tan 315°$ b) $\sin\frac{5\pi}{8}$ c) $\sec\left(\frac{-\pi}{10}\right)$ ↳ $\frac{1}{\cos\left(\frac{-\pi}{10}\right)}$ d) $\cot\left(\frac{-7\pi}{6}\right)$



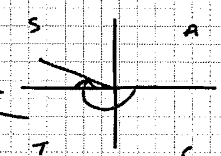
NEGATIVE



POSITIVE



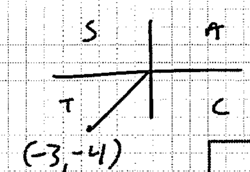
POSITIVE



NEGATIVE

(3) Determine the exact value of the remaining 5 trig ratios given:

$\tan\theta = \frac{4}{3}$ in quadrant III



$r = \sqrt{9+16}$

$r = 5$

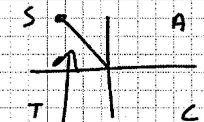
$\boxed{x = -3 \quad y = -4}$

$$\boxed{\begin{array}{lll} \sin\theta = \frac{-4}{5} & \cos\theta = \frac{-3}{5} & \\ \csc\theta = -\frac{5}{4} & \sec\theta = -\frac{5}{3} & \cot\theta = \frac{3}{4} \end{array}}$$

(4) Given $\csc\theta = \frac{4}{3}$ in quadrant II find $\cos\theta$

$r = 4$

$y = 3$



$\longrightarrow \sin\theta = \frac{3}{4}$

$\boxed{x \text{ must be negative}}$

$x^2 + y^2 = r^2$

$x \overset{*}{=} (-)\sqrt{16-9}$

$\underline{\underline{x = -\sqrt{7}}}$

$$\boxed{\cos\theta = \frac{-\sqrt{7}}{4}}$$

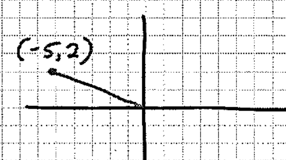
(5) Determine $\sin\theta$ and $\cos\theta$ if θ is an angle in standard position whose terminal side is the graph $2x + 5y = 0$, $x \leq 0$.

$5y = -2x$

$y = \frac{-2x}{5}$

$\frac{-2}{5}$ OR $\boxed{\frac{2}{-5} \frac{\text{Rise}}{\text{Run}}}$

$\underline{x \leq 0}$



$x^2 + y^2 = r^2$

$25 + 4 = r^2$

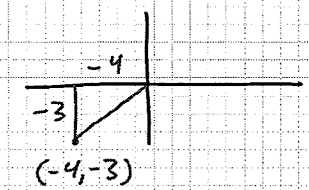
$\sqrt{29} = r$

$$\boxed{\sin\theta = \frac{2}{\sqrt{29}} \qquad \cos\theta = \frac{-5}{\sqrt{29}}}$$

⑥ Determine $\tan\theta$ and $\sec\theta$ if θ is an angle in standard position whose terminal side is the graph

$3x - 4y = 0,\ y \leq 0.$

℞ 220
#2,3,5-8
(a,c,e)



$-4y = -3x$

$y = \frac{-3}{-4}x$

$y \leq 0$

$x^2 + y^2 = r^2$

$(-4)^2 + (-3)^2 = r^2$

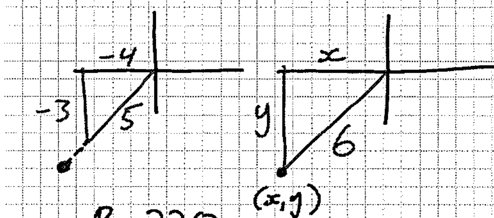
$25 = r^2$

$5 = r$

$$\boxed{\tan\theta = \frac{3}{4} \qquad \sec\theta = \frac{5}{-4}}$$

⑦ Determine the coordinates of the point at the given distance from the origin in the stated quadrant

$6,\ \text{III},\ \tan\theta = \frac{3}{4}$



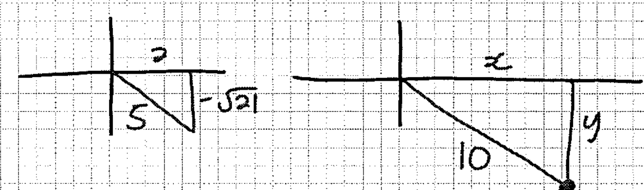
$\frac{x}{-4} = \frac{6}{5}$ $\qquad$ $\frac{y}{-3} = \frac{6}{5}$

$x = \frac{-24}{5}$ $\qquad$ $y = \frac{-18}{5}$

$$\boxed{\left(\frac{-24}{5}, \frac{-18}{5}\right)}$$

℞ 220
#2,3,5-8
(a,c,e)

b) $10,\ \text{IV},\ \cos\theta = \frac{2}{5}$



$\frac{x}{2} = \frac{10}{5}$ $\qquad$ $\frac{y}{-\sqrt{21}} = \frac{10}{5}$

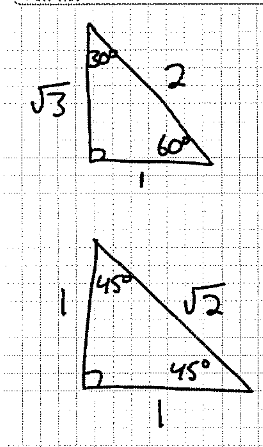
$\underline{\underline{x = 4}}$ $\qquad$ $\underline{\underline{y = -2\sqrt{21}}}$

$$\boxed{(4, -2\sqrt{21})}$$

℞ 220
#2,3,5-8
(a,c,e)

Note Title 2012-11-14

5.3 Trig Functions – Special Angles



$\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$

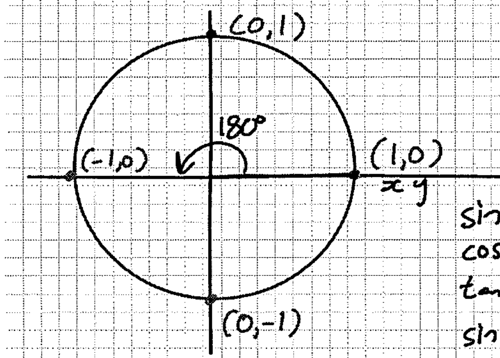
$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$ $\tan 60° = \sqrt{3}$

$\sin 45° = \frac{1}{\sqrt{2}}$ $30° = \frac{\pi}{6}$ $45° = \frac{\pi}{4}$

$\cos 45° = \frac{1}{\sqrt{2}}$ $60° = \frac{\pi}{3}$ $90° = \frac{\pi}{2}$

$\tan 45° = 1$



QUADRANTAL ANGLES

0°, 90°, 180°, 270°, 360°

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$\sin 0° = 0$ $\sin 180° = 0$

$\cos 0° = 1$ $\cos 180° = -1$

$\tan 0° = 0$ $\tan 180° = 0$

$\sin 90° = 1$ $\sin 270° = -1$

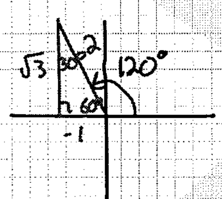
$\cos 90° = 0$ $\cos 270° = 0$

$\tan 90° =$ undefined $\tan 270° =$ undefined

$$\boxed{\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}}$$

① Determine the exact primary trig ratios for 120°

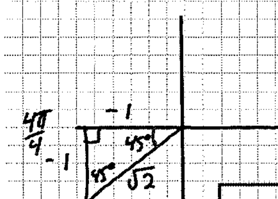
$\sin 120°$ $\cos 120°$ $\tan 120°$



$$\boxed{\sin 120° = \frac{\sqrt{3}}{2} \quad \cos 120° = -\frac{1}{2} \quad \tan 120° = -\sqrt{3}}$$

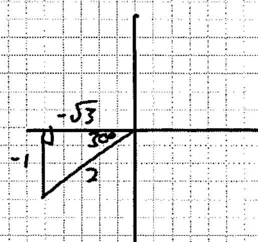
① Determine the exact value

a) $\cos \frac{5\pi}{4}$ b) $\csc 210°$



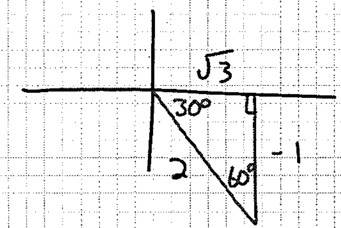
$\frac{\pi}{4} = 45°$

$$\boxed{\cos \frac{5\pi}{4} = \frac{-1}{\sqrt{2}} \text{ OR } -\frac{\sqrt{2}}{2}}$$



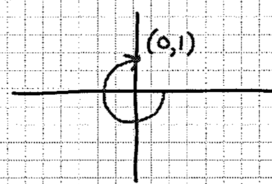
$$\boxed{\csc 210° = -2}$$

c) $\cot\left(-\frac{\pi}{6}\right)$



$$\boxed{\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}}$$

d) $\sec\left(-\frac{3\pi}{2}\right)$

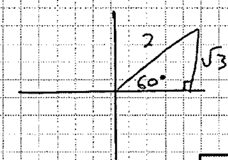


$\sec\theta = \frac{r}{x} = \frac{1}{0}$

$\boxed{\text{undefined}}$

e) $\sin^2\left(\frac{\pi}{3}\right)$

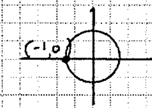
$\left[\sin\left(\frac{\pi}{3}\right)\right]^2$

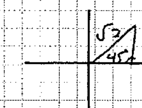


$\left(\frac{\sqrt{3}}{2}\right)^2 = \boxed{\frac{3}{4}}$

f) $\cos^2(\pi) - \sin^2\left(\frac{\pi}{4}\right)$

$[\cos(\pi)]^2 - \left[\sin\left(\frac{\pi}{4}\right)\right]^2$



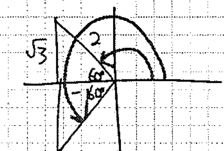


$(-1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$1 - \frac{1}{2} = \boxed{\frac{1}{2}}$

② Solve for x, $0 \leq x < 2\pi$ (Must be in radians)

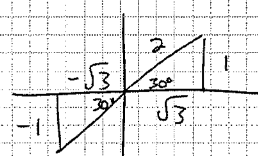
a) $\cos x = -\frac{1}{2}$



$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

120°, 240°

b) $\cot x = \sqrt{3}$

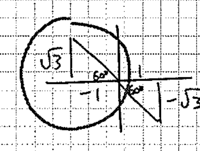


$$\boxed{x = \frac{\pi}{6}, \frac{7\pi}{6}}$$

30° 210°

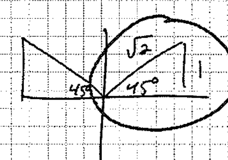
③ Find the smallest positive θ for which:

a) $\tan\theta = -\sqrt{3}$



$$\boxed{\theta = 120° \text{ OR } \frac{2\pi}{3}}$$

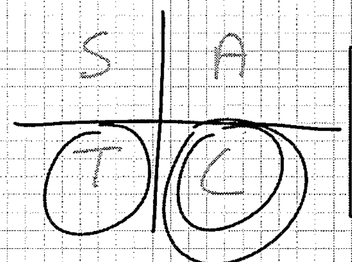
b) $\csc\theta = \sqrt{2}$



$$\boxed{\theta = 45° \text{ OR } \frac{\pi}{4}}$$

(4) Given $\sin\theta = \frac{-1}{\sqrt{2}}$ and $\cos\theta = \frac{1}{\sqrt{2}}$

a) In what quadrant does the terminal arm of $\angle\theta$ lie?



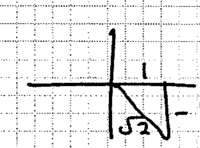
$\boxed{\text{IV}}$

b) What is the exact value of $\tan\theta$?

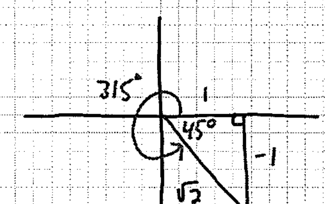
$\sin\theta = \frac{-1}{\sqrt{2}}$ $\quad \cos\theta = \frac{1}{\sqrt{2}}$

$y = -1$
$r = \sqrt{2}$
$x = 1$

$\tan\theta = \frac{y}{x} = \frac{-1}{1} = \boxed{-1}$



c) What is the reference angle and principal angle?



Reference $\angle = 45°$ OR $\frac{\pi}{4}$

Principal $\angle = 315°$ OR $\frac{7\pi}{4}$

Pg 231
*1, 2, 5, 6 (a, c, e ...)
7c)
11 (a, c, e, g)

Pg 231
*1, 2, 5, 6 (a, c, e ...)
7c)
11 (a, c, e, g)

5.2/5.3 Review

Note Title 2014-04-24

① Determine the exact value of the remaining 5 trig ratios given:

$\tan\theta = \frac{5}{-2}$ and θ is in quadrant IV.

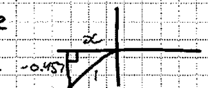
$r = \sqrt{4+25} = \sqrt{29}$

$$\boxed{\begin{array}{l} \sin\theta = \frac{-5\sqrt{29}}{\sqrt{29}\sqrt{29}} = \frac{-5\sqrt{29}}{29} \quad \cos\theta = \frac{2\sqrt{29}}{\sqrt{29}\sqrt{29}} = \frac{2\sqrt{29}}{29} \\ \csc\theta = \frac{\sqrt{29}}{5} \quad \sec\theta = \frac{\sqrt{29}}{2} \quad \cot\theta = -\frac{2}{5} \end{array}}$$

② Find $\cos\theta$ if $\sin\theta = -0.457$ and $\tan\theta > 0$.
(3 decimal places)

$\sin\theta = \frac{-0.457}{1}$ $\quad y = -0.457$, $r = 1$

* sin −ve
tan +ve
in III



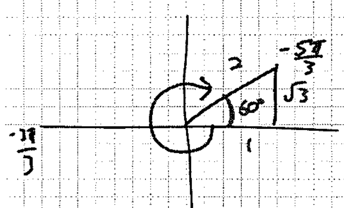
III $\therefore$ $\cos\theta < 0$

$x^2 + y^2 = r^2 \quad x = (-)\sqrt{1-(-0.457)^2}$

$$\boxed{\cos\theta = -0.889}$$

③ Determine the exact value

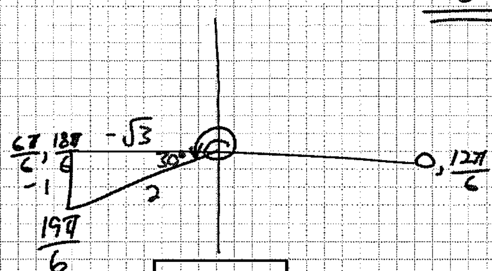
a) $\sin\left(-\frac{5\pi}{3}\right)$



$$\boxed{\frac{\sqrt{3}}{2}}$$

b) $\csc\left(\frac{19\pi}{6}\right)$ * Co-Terminal ∠

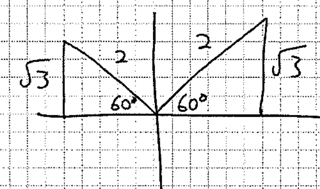
$\frac{19\pi}{6} - \frac{12\pi}{6} = \frac{7\pi}{6}$



$$\boxed{-2}$$

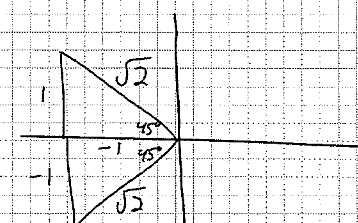
④ Solve for θ where $0 \leq \theta < 2\pi$

a) $\sin\theta = \frac{\sqrt{3}}{2}$



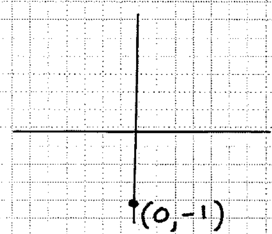
$$\boxed{\frac{\pi}{3}, \frac{2\pi}{3}}$$

b) $\sec\theta = -\sqrt{2}$



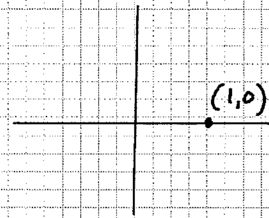
$$\boxed{\frac{3\pi}{4}, \frac{5\pi}{4}}$$

c) $\sin\theta = -1$



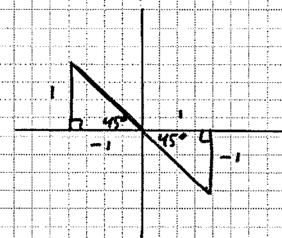
$\boxed{\frac{3\pi}{2}}$

d) $\cos\theta = 1$



$\boxed{\begin{array}{c} 0 \\ 0 \le \theta < 2\pi \end{array}}$

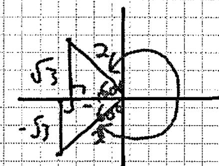
e) $\tan\theta = -1$



$\boxed{\frac{3\pi}{4}, \frac{7\pi}{4}}$

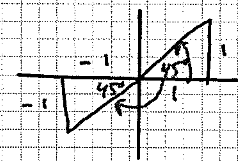
⑤ Solve for θ where $-\pi \le \theta \le \pi$

a) $\cos\theta = -\frac{1}{2}$



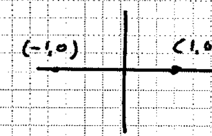
$\boxed{\theta = \frac{2\pi}{3}, -\frac{2\pi}{3}}$

b) $\tan\theta = 1$



$\boxed{\theta = \frac{\pi}{4}, -\frac{3\pi}{4}}$

c) $\sin\theta = 0$



$\boxed{\theta = 0, \pi, -\pi}$

*WORKSHEET

Pg 220
\#2,3,5,6
(b,d)

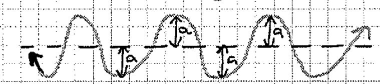
Pg 232
\#5,6,11
(b,d,f)

Note Title 2014-04-28

5.4 Pt.1 Graphing Sine and Cosine

Given: $y = a\sin(bx)$ OR $y = a\cos(bx)$

AMPLITUDE: (a)



$$\text{amplitude} = \frac{|\text{max} - \text{min}|}{2} \leftarrow \text{"absolute value"}$$

(Always Positive)

ex. $y = -2\sin x$ amplitude $= +2$

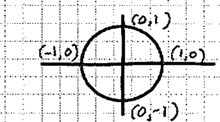
PERIOD: (T) One complete cycle

FOR SINE + COSINE: $T = \frac{2\pi}{b}$ OR $T = \frac{360°}{b}$

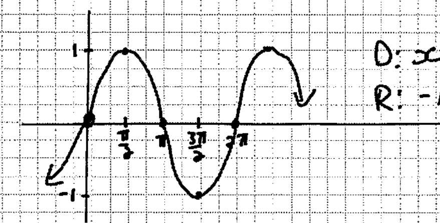
① Graph using a table of values (use quadrantal angles)

a) $y = \sin\theta$

"Use unit circle": $\sin\theta = \frac{y}{r}$



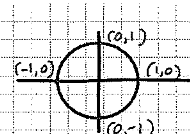
θ	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



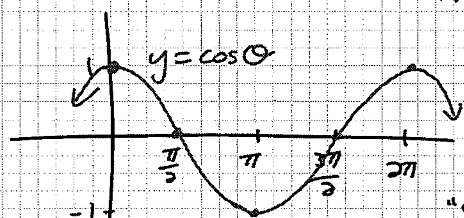
D: $x \in \mathbb{R}$
R: $-1 \leq y \leq 1$

b) $y = \cos\theta$

$\cos\theta = \frac{x}{r}$

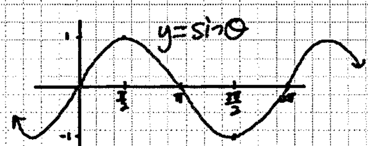


θ	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



D: $x \in \mathbb{R}$
R: $-1 \leq y \leq 1$

"SINUSOIDAL"



$y = a\sin b(x - c) + d \leftarrow$ vertical displacement moves up/down

- a: amplitude
- b: changes period, $T = \frac{2\pi}{b}$
- c: phase shift moves left/right

② Given $y = 3\sin\left(2x - \frac{\pi}{2}\right) + 2$

a) Determine

i) amplitude: ii) period: iii) phase shift:

* ALWAYS FACTOR FIRST IF POSSIBLE *

$$\boxed{y = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 2}$$

$\boxed{amp = 3}$ $T = \frac{\cancel{2}\pi}{\cancel{2}} = \boxed{T = \pi}$ $\boxed{\text{Right } \frac{\pi}{4}}$

iv) Vertical Displacement

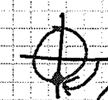
$\boxed{\text{Up } 2}$

v) y-intercept

$x = 0$

$y = 3\sin\left(2x - \frac{\pi}{2}\right) + 2$

$y = 3\sin\left(-\frac{\pi}{2}\right) + 2$



$(0, \underline{\underline{-1}})$

$y = 3(-1) + 2$

$\boxed{y = -1}$

vi) Domain + Range

$y = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 2$

D: $x \in \mathbb{R}$

Range: $-1 \leq y \leq 1$

$(\times 3)$ $-3 \leq y \leq 3$

$(+2)$ R: $-1 \leq y \leq 5$

b) Graph by mapping points $y = 3\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 2$

C - Vert Exp $\times 3$, Horiz Compr. $\times \frac{1}{2}$

R -

T - Right $\frac{\pi}{4}$, Up 2

$(x, y) \rightarrow \left(\frac{1}{2}x + \frac{\pi}{4}, 3y + 2\right)$

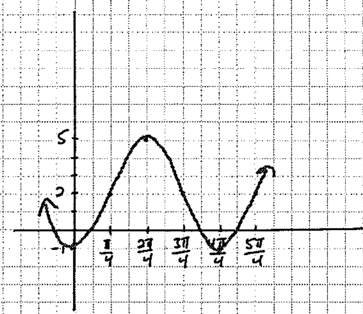
$y = \sin x$

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$\longrightarrow$

$y = 3\sin 2(x - \frac{\pi}{4}) + 2$ $\quad (\frac{1}{2}x + \frac{\pi}{4}, 3y+2)$

x	y
$\frac{\pi}{4}$	2
$\frac{2\pi}{4}$	5
$\frac{3\pi}{4}$	2
$\frac{4\pi}{4}$	-1
$\frac{5\pi}{4}$	2



(3) Given $y = \frac{1}{2}\cos(2x - \frac{\pi}{2}) - 2$ determine:

a) Period, Phase Shift, Domain, Range, and y-intercept

$$y = \frac{1}{2}\cos(2(x - \frac{\pi}{4})) - 2$$

$T = \frac{2\pi}{2}$

$T = \pi$

P.S. Right $\frac{\pi}{4}$

D: $x \in \mathbb{R}$

R: usually $-1 \le y \le 1$

$-\frac{1}{2} \le y \le \frac{1}{2}$

R: $-2.5 \le y \le -1.5$

$y = \frac{1}{2}\cos(-\frac{\pi}{2}) - 2$

$y = \frac{1}{2}(0) - 2$

$y = -2$

(3) Graph $y = \frac{1}{2}\cos(3x - \frac{\pi}{2}) - 2$

$y = \frac{1}{2}\cos(3(x - \frac{\pi}{6})) - 2$

$(x,y) \to (\frac{1}{3}x + \frac{\pi}{6}, \frac{1}{2}y - 2)$

C - vert compr. $\times \frac{1}{2}$

- horiz compr. $\times \frac{1}{3}$

R -

T - Right $\frac{\pi}{6}$, Down 2

$y = \cos x$

$(0,1)$

$(\frac{\pi}{2}, 0)$

$(\pi, -1)$

$(\frac{3\pi}{2}, 0)$

$(2\pi, 1)$

$\longrightarrow$

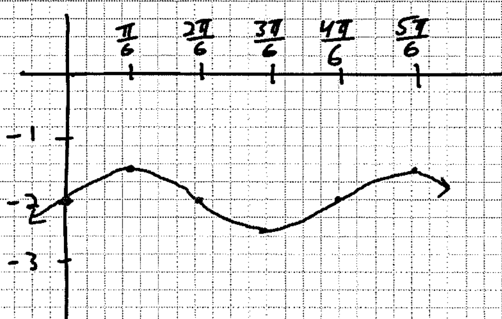
$(\frac{\pi}{6}, -1.5)$

$(\frac{2\pi}{6}, -2)$

$(\frac{3\pi}{6}, -2.5)$

$(\frac{4\pi}{6}, -2)$

$(\frac{5\pi}{6}, -1.5)$



$(\frac{1}{3}x + \frac{\pi}{6}, \frac{1}{2}y - 2)$

$(\frac{\pi}{6}, -1.5)$

$(\frac{\pi}{3}, -2)$

$(\frac{\pi}{2}, -2.5)$

$(\frac{2\pi}{3}, -2)$

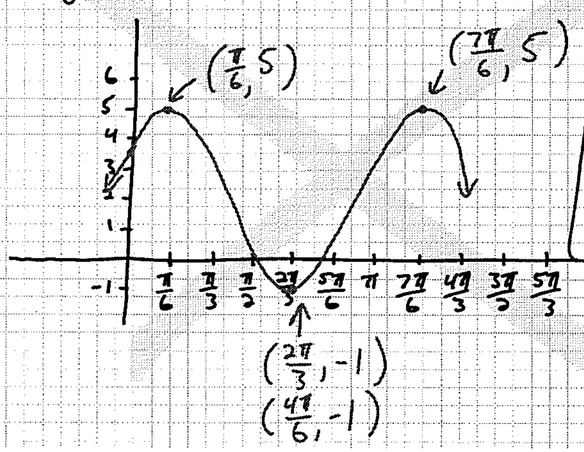
$(\frac{5\pi}{6}, -1.5)$

Pg 242

1, 3, 6, 7

④ Determine the equation of the graphs in the form

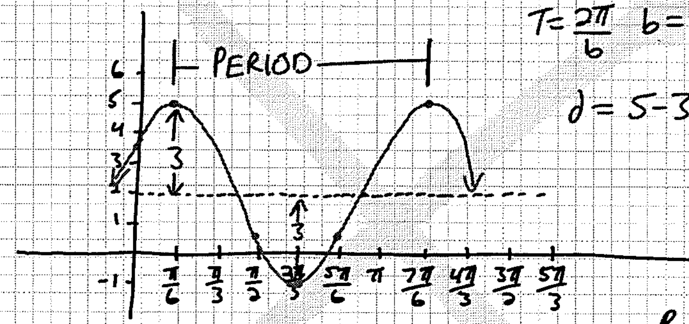
$y = a\cos b(x-c) + d$



$a = \dfrac{|\text{max} - \text{min}|}{2}$

$T = \dfrac{2\pi}{b} \therefore b = \dfrac{2\pi}{T}$

$* \; d = \text{Max} - a$



$a = 3$

$T = \dfrac{2\pi}{b} \quad b = \dfrac{2\pi}{T} = \dfrac{2\pi}{\pi} = 2$

$d = 5 - 3 \quad d = 2$

Phase Shift: Right $\frac{\pi}{6}$

$\therefore \boxed{c = \frac{\pi}{6}}$

$\boxed{y = 3\cos\left(2\left(x - \frac{\pi}{6}\right)\right) + 2}$

Pg 242

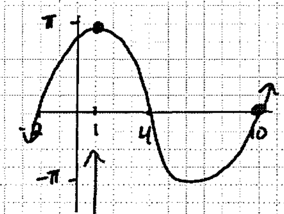
* 1, 3, 6, 7

5.4 Pt.2

Note Title 2013-11-25

① Pg 243 #5e) Determine the equation given the graph:

$a = \pi \quad T = \frac{2\pi}{b} \quad b = \frac{2\pi}{T} \quad \therefore b = \frac{2\pi}{12} = \frac{\pi}{6}$



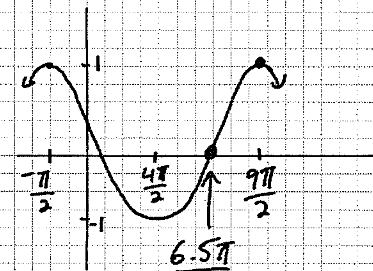
Half-way between -2 and 4

$$\boxed{\begin{aligned} y &= \pi \sin \tfrac{\pi}{6}(x-10) \\ y &= \pi \sin \tfrac{\pi}{6}(x+2) \\ y &= \pi \cos \tfrac{\pi}{6}(x-1) \end{aligned}}$$

$* \; d = Max - a$

③ Pg 243 #5g)

$a = 1, \; T = \frac{10\pi}{2} = 5\pi \therefore b = \frac{2\pi}{5\pi} \cdot \frac{2}{5}$

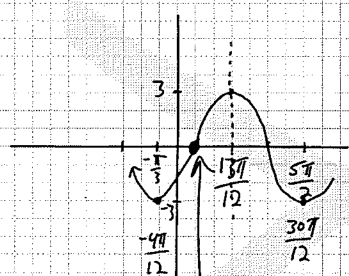


$\frac{6.5\pi}{2}$

$\frac{13\pi}{4}$

$$\boxed{\begin{aligned} y &= \sin \tfrac{2}{5}\left(x - \tfrac{13\pi}{4}\right) \\ y &= \cos \tfrac{2}{5}\left(x - \tfrac{9\pi}{2}\right) \\ y &= \cos \tfrac{2}{5}\left(x + \tfrac{\pi}{2}\right) \end{aligned}}$$

⑤ h)



$T = \frac{17\pi}{6}$

$-\frac{4\pi}{12} + \frac{17\pi}{24}$

$-\frac{8\pi}{24} + \frac{17\pi}{24} = \frac{9\pi}{24} = \frac{3\pi}{8}$

$\therefore$ Right $\frac{3\pi}{8}$

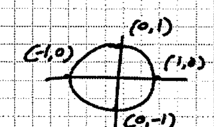
GRAPHING TAN

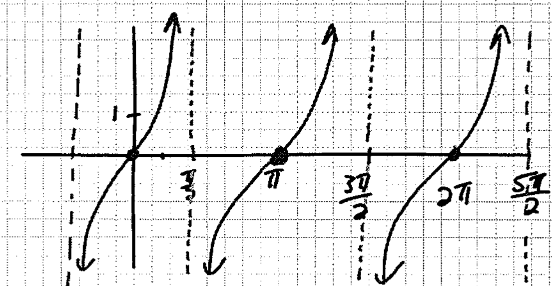
Note Title 2012-11-19

③ Graph $y = \tan\theta$ and determine the domain and range

θ	y
0	0
$\frac{\pi}{2}$	undefined
π	0
$\frac{3\pi}{2}$	undefined
2π	0

$\tan\theta = \frac{y}{x}$

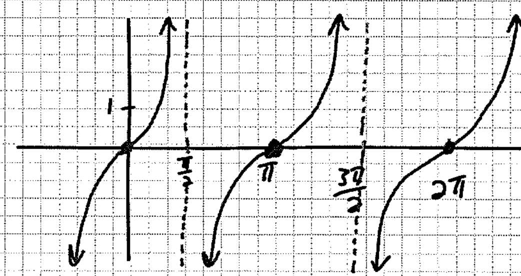




$\boxed{T = \frac{\pi}{b}}$

$\tan\theta = \frac{y}{x}$

θ	y
0	0
$\frac{\pi}{2}$	— undefined
π	0
$\frac{3\pi}{2}$	— undefined
2π	0



$\boxed{T = \frac{\pi}{b}}$

D: $x \neq \frac{n\pi}{2}$
where n is an odd integer

OR

D: $x \neq (2n+1)\frac{\pi}{2}$

$n \in I$

R: $y \in R$

④ Determine the period, domain, range, and graph

$y = -4\tan\frac{1}{2}x$

Vert. Exp × 4

Horiz. Exp × 2

Reflected across x-axis

$(x, y) \rightarrow (2x, -4y)$

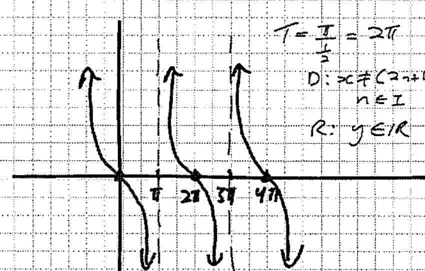
* Falls to Right

$y = \tan x$

x	y
0	0
$\frac{\pi}{2}$	—
π	0
$\frac{3\pi}{2}$	—
2π	0

$\rightarrow$

x	y
0	0
π	—
2π	0
3π	—
4π	0



$T = \frac{\pi}{\frac{1}{2}} = 2\pi$

D: $x \neq (2n+1)\pi$
$n \in I$

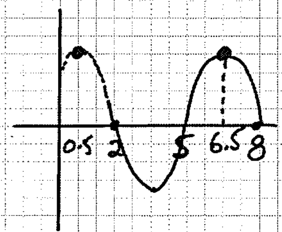
R: $y \in R$

Pg 242

* 4

5

5 b)



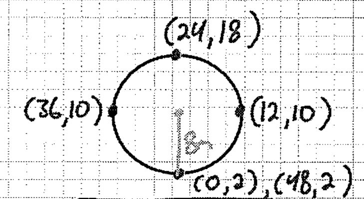
$T=6$

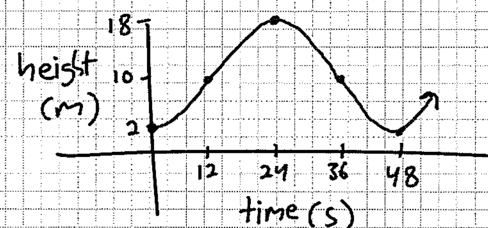
Note Title 2008-04-02

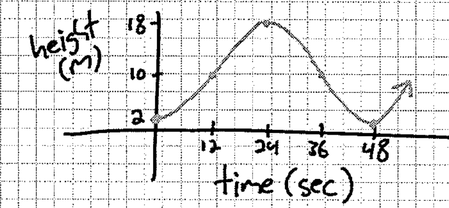
5.5 Applications

① The loading platform of a Ferris wheel is 2m above ground level. The Ferris wheel has a radius of 8m and completes one revolution in 48 seconds.

a) Sketch a graph and determine a sinusoidal function that describes the ride.







$a = \frac{|18-2|}{2} = 8$ $b = \frac{2\pi}{48} = \frac{\pi}{24}$ "Max − a" $d = 18-8$

$d = 10$

Phase Shift: Sine − Right 12

Cosine − Right 24 OR None but reflected in x-axis

$$y = 8\sin\frac{\pi}{24}(x-12)+10 \qquad y = 8\cos\frac{\pi}{24}(x-24)+10$$

$$y = -8\cos\frac{\pi}{24}x+10$$

SINE

Right 12
$a = 8$
$d = 10$
$T = 48$
$b = \frac{\pi}{24}$

$$y = 8\sin\frac{\pi}{24}(x-12)+10$$

b) Determine the riders height 63 seconds after getting on the Ferris wheel.

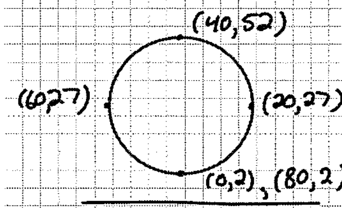
$$y = 8\cos\left(\left(\pi/24\right)(63-24)\right)+10$$

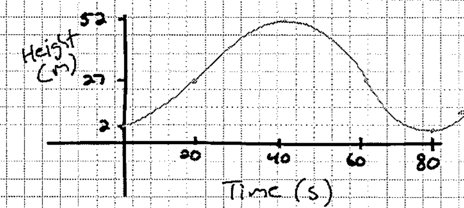
* RADIANS MODE

$$\boxed{13.06\text{ m}}$$

② A Ferris wheel has a diameter of 50m and rotates once every 80 seconds. A rider enters the seat at the lowest point of the Ferris wheel 2 metres above the ground.

Determine a sine function that gives the height, h, after t seconds of motion for the rider.





$a = 25$

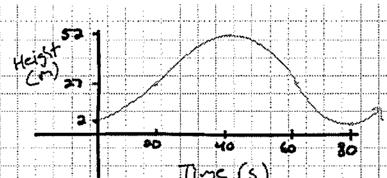
$T = 80$

$\therefore\ b = \frac{2\pi}{80}$

$b = \frac{\pi}{40}$

P.S. = Right 20

d = up 27



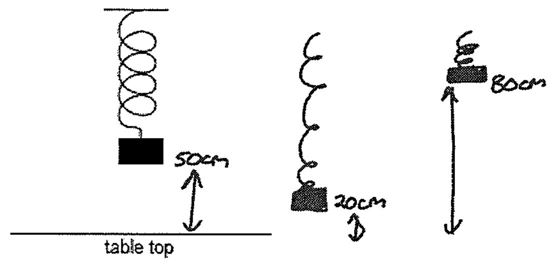
$$\boxed{h = 25\sin\left(\frac{\pi}{40}(t-20)\right)+27}$$

Cosine: $h = -25\cos\left(\frac{\pi}{40}t\right)+27$

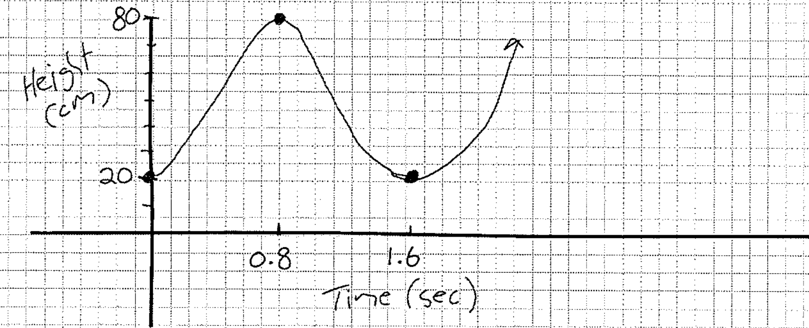
OR

$h = 25\cos\left(\frac{\pi}{40}(t-40)\right)+27$

(3) A mass is supported by a spring so that it rests 50 cm above a table top, as shown in the diagram below. The mass is pulled down to a height of 20 cm above the table top and released at time $t = 0$. It takes 0.8 seconds for the mass to reach a maximum height of 80 cm above the table top. As the mass moves up and down, its height h, in cm, above the table top, is approximated by a sinusoidal function of the elapsed time t, in seconds, for a short period of time.



Determine an equation for a sinusoidal function that gives h as a function of t. **(4 marks)**



$a = \frac{80-20}{2} = 30$

$T = 1.6 \quad b = \frac{2\pi}{1.6}$ OR $\frac{20\pi}{16} = \frac{5\pi}{4}$

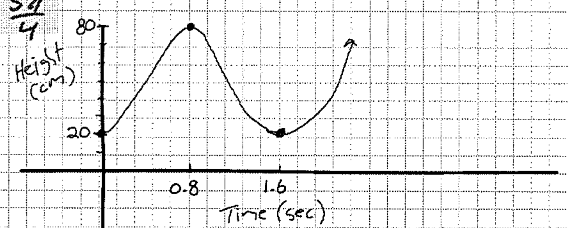
$d = 80 - 30 = 50$

P.S.: sin: Right 0.4

cos: Right 0.8

OR

No Phase Shift

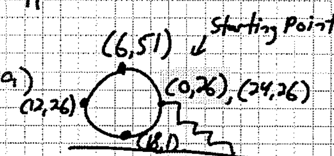


$$\boxed{h = 30\sin\frac{5\pi}{4}(t-0.4)+50 \text{ OR } h = 30\cos\frac{5\pi}{4}(t-0.8)+50 \text{ OR } h = -30\cos\frac{5\pi}{4}t+50}$$

Pg 247 #2, 9(a,b), 11

Review: Pg 250 #1-39 *Not 18, 35, 37

(9) a)



Note Title 2008-04-08

Chp 5 Review

① Graph $y=3\sin(2x-\frac{2\pi}{3})+2$ by mapping points.

$*\ y=3\sin 2(x-\frac{\pi}{3})+2 \qquad (x,y)\to(\frac{1}{2}x+\frac{\pi}{3}, 3y+2)$

$y=\sin x$

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

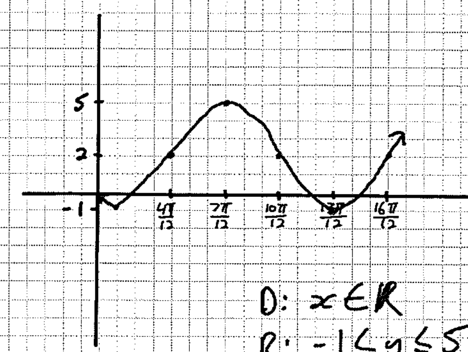
$\longrightarrow$

x	y
$\frac{4\pi}{12}$	2
$\frac{7\pi}{12}$	5
$\frac{10\pi}{12}$	2
$\frac{13\pi}{12}$	-1
$\frac{16\pi}{12}$	2

$\longrightarrow \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right)+\frac{\pi}{3}$

$\frac{\pi}{4}+\frac{\pi}{3}$

$\frac{3\pi}{12}+\frac{4\pi}{12}=\frac{7\pi}{12}$



D: $x\in\mathbb{R}$

R: $-1\le y\le 5$

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

$\longrightarrow$

x	y
$\frac{4\pi}{12}$	2
$\frac{7\pi}{12}$	5
$\frac{10\pi}{12}$	2
$\frac{13\pi}{12}$	-1
$\frac{16\pi}{12}$	2

② Determine the range and vertical displacement:

a) $y=-3\sin 2x+4$

usually $-1\le y\le 1$

Multiply by 3 then add 4

$-3^{+4}\le y\le 3^{+4}$

$\boxed{1\le y\le 7}$

$\boxed{\text{Up } 4}$

b) $y=d\cos(x-e)-3d$

Multiply by d then subtract $3d$

$-d\le y\le d$

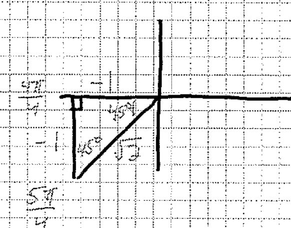
$-d-3d\le y\le d-3d$

$\boxed{-4d\le y\le -2d}$

$\boxed{\text{Down } 3d}$

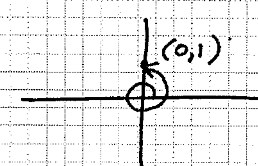
③ Determine the exact value:

a) $\sec\frac{5\pi}{4}$



$\sec\frac{5\pi}{4}=-\sqrt{2}$

b) $\sin 450°$



$\sin 450°=1$

(4) $y = \sin\left(\frac{\pi}{2}x + \pi\right)$ compared to $y = \sin\frac{\pi}{2}x$ has a:

A. horizontal translation to the right π units

B. horizontal translation to the right 2 units

C. horizontal translation to the left π units

D. horizontal translation to the left 2 units

$\pi \div \frac{\pi}{2}$

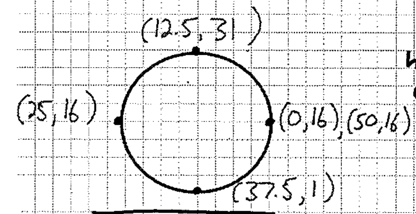
$\pi \cdot \frac{2}{\pi}$

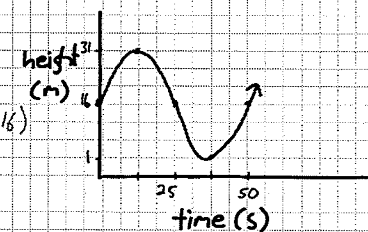
$\underline{\underline{2}}$

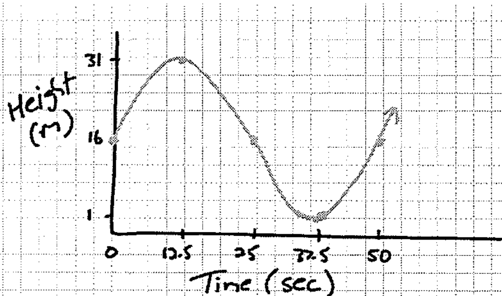
$y = \sin\left(\frac{\pi}{2}(x+2)\right)$ $\therefore$ $\boxed{\text{Left } 2 \quad \text{D}}$

(5) A Ferris wheel has a diameter of 30m. The wheel makes one complete rotation in 50 seconds. Passengers get on the ride on the side, 16m above the ground. The bottom of the Ferris wheel is 1m above ground.

a) Write a sine function to describe the height of one passenger in metres who gets on the ride as a function of time in seconds.







$a = \frac{31-1}{2} = 15$

$b = \frac{2\pi}{T} = \frac{2\pi}{50} = \frac{\pi}{25}$

$c = 0$

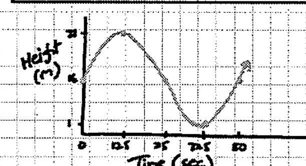
$d = Max - a$
$= 31 - 15 = 16$

$$\boxed{h(t) = 15\sin\frac{\pi}{25}t + 16}$$

b) What height will the passenger be at after 15 seconds?

$t = 15 \rightarrow h = 15\sin\left(\left(\frac{\pi}{25}\right)\cdot 15\right) + 16$

$$\boxed{h = 30.27m}$$



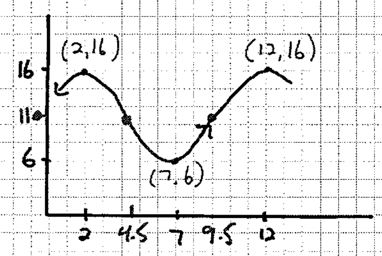
Pg 250 #1-39
*Not 18, 35, 37

Note Title

Chp 5 Review Day 2

2014-12-10

① A sine curve has a maximum point at (2, 16) and the nearest minimum point to the right is at (7, 6). Determine a possible equation for this curve.



$a = \frac{16-6}{2} = 5$

$b = \frac{2\pi}{10} = \frac{\pi}{5}$

$-\sin$

$\downarrow$

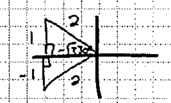
$c = -0.5, 4.5, 9.5\ldots$

$d = 16 - 5 = 11$

$$\boxed{y = 5\sin\frac{\pi}{5}(x - 9.5) + 11}$$

② Determine θ given $0 \leq \theta < 2\pi$

a) $\sec\theta = -\frac{2}{\sqrt{3}}$



$\theta = 150°, 210°$

$$\boxed{\theta = \frac{5\pi}{6}, \frac{7\pi}{6}}$$

b) $\tan\theta = -1$

$\theta = 135°, 315°$

$$\boxed{\theta = \frac{3\pi}{4}, \frac{7\pi}{4}}$$

c) $\sin\theta = 0$

(-1,0) (1,0)

$$\boxed{\theta = 0, \pi}$$

Pg 250 #1-39

*Not 18, 35, 37