Chapter 5 Review

Section 5.1 Extra Practice

1. a) Sketch the graph of $y = \sin\theta$ for $-360° \le \theta \le 360°$. Identify the key points by labelling their coordinates on the graph.
   b) What is the exact value of this function at 225°?
   c) What are the x-intercepts of the graph?

2. a) Sketch the graph of $y = \cos x$ for $0 \le x \le 2\pi$.
   b) What is the exact value of this function at $\frac{4\pi}{3}$?
   c) What is the minimum value of this function?
   d) What is the y-intercept of this function?

3. a) Sketch the graph of $y = 4\sin x$ for $x \in R$.
   b) State the range of the function.
   c) What is the period of the function in radians?
   d) State the amplitude.

4. a) Sketch the graph of $y = -\frac{1}{4}\cos\theta$ for $\theta \in R$.
   b) State the coordinates of the y-intercept.
   c) State the range of the function.
   d) State the amplitude.

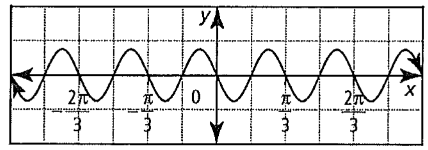
5. a) Sketch the graph of $y = \sin 3x$ for $0° \le x \le 360°$. Clearly plot the key points.
   b) What is the period of the function, in degrees?
   c) What is the range of this function?
   d) State the amplitude.

6. a) Sketch the graph of $y = \cos\frac{1}{2}x$, in radians. Show one complete cycle.
   b) State the coordinates of the y-intercept.
   c) What is the period of this function?
   d) State the amplitude.

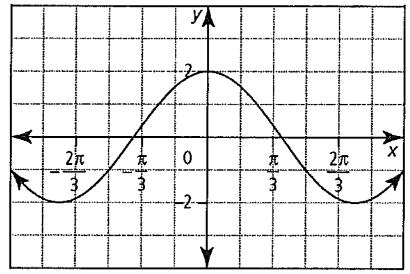
7. For each function, state the amplitude. Then, state the period in degrees and radians.
   a) $y = 4\sin 2x$ b) $y = -3\cos\frac{1}{5}x$
   c) $y = \frac{2}{3}\sin\frac{2}{3}x$ d) $y = -\frac{1}{4}\cos(-3x)$

8. Describe how each function's graph is related to the graph of $y = \cos x$.
   a) $y = 2\cos 4x$ b) $y = -\cos\frac{1}{5}x$
   c) $y = -3\cos\frac{5}{2}x$ d) $y = 5\cos(-x)$

9. Determine the amplitude & period for the graphs below.
   a)



   b)



Section 5.2 Extra Practice

1. Graph each pair of functions on the same grid. For each, clearly plot the key points.
   a) $y = 2\sin x$ and $y = 2\sin(x + 45°) - 3$
   b) $y = \cos 3x$ and $y = \cos 3\left(x - \frac{\pi}{2}\right) + 1$
   c) $y = -\sin\frac{1}{2}x$ and $y = -\sin\frac{1}{2}\left(x + \frac{\pi}{4}\right) - 2$
   d) $y = -3\cos x$ and $y = -3\cos(x + 60°) - 4$

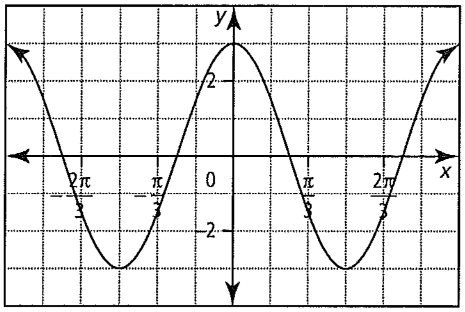
2. For each function, determine the phase shift and vertical displacement with respect to $y = \cos x$.
   a) $y = 0.15\cos 2(x - 25°) + 3.2$
   b) $y = -2\cos 3\left(x + \frac{\pi}{6}\right) - 7$
   c) $y = \cos\left(2x - \frac{\pi}{4}\right) + 5$ d) $y = 6\cos(3x + 2\pi) - 1$

3. Determine the period and range for each function.
   a) $y = 4\sin 2(x + 30°) - 6$ b) $y = -3\sin\frac{1}{3}\left(x + \frac{\pi}{3}\right) + 2$
   c) $y = 2.3\sin(5x - 30°) + 4.2$ d) $y = -7\sin\left(3x + \frac{\pi}{2}\right) - 3$

4. Determine the period & range of $y = a\cos b(x - c) + d$.

5. Given the following characteristics, write each equation in the form $y = a\sin b(x - c) + d$.
   a) phase shift of $\frac{\pi}{2}$, period of $\frac{\pi}{2}$, vertical displacement of 5, and amplitude of 3
   b) period of 120°, phase shift of −50°, amplitude of $\frac{1}{2}$, and vertical displacement of −4
   c) period of 8π and phase shift of $\frac{\pi}{2}$
   d) period of 3π and vertical displacement of 2

6. Consider the graph of $y = 3 \cos 2x$.



Write the equation of this graph as a sine function that has undergone a phase shift left.

7. For the given graph, determine

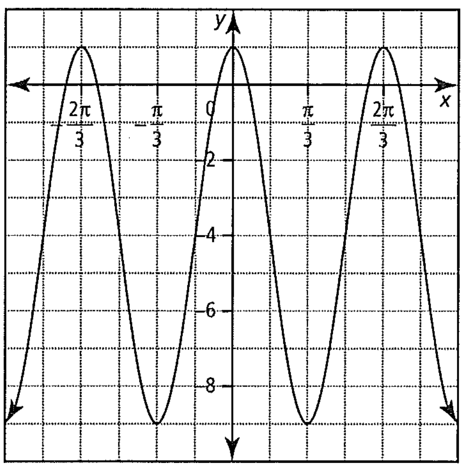
a) the amplitude **b)** the vertical displacement

c) the period

d) its equation in the form $y = a \cos b(x - c) + d$

e) the maximum value of y, and the values of x for which it occurs over the interval $0 \le x \le 2\pi$

f) the minimum value of y, and the values of x for which it occurs over the interval $0 \le x \le 2\pi$



8. Determine an equation of the sine curve with a minimum point at (90°, 4) and its nearest maximum to the right at (120°, 10).

Section 5.3 Extra Practice

1. Let $y = \tan \theta$ for $0 \le \theta \le 2\pi$. State the values for θ when

a) $y = 0$ **b)** $y = 1$ **c)** $y = -1$ **d)** y is undefined

2. For $y = \tan x$, state the exact value of y for each.

a) $x = 30°$ **b)** $x = 45°$ **c)** $x = 60°$

d) $x = 90°$ **e)** $x = 120°$ **f)** $x = 135°$

g) $x = 150°$ **h)** $x = 180°$

3. a) Graph $y = \tan x$ for $0° \le x \le 360°$.

b) State the domain. **c)** State the range.

d) State the period.

4. a) Graph $y = \tan x$ for $-\pi \le x \le \pi$.

b) State the coordinates of the x-intercepts.

c) State the equations of the asymptotes.

d) What is the y-intercept?

5. Does $y = \tan x$ have an amplitude? Explain.

6. State the asymptotes and domain of $y = \tan x$, in degrees.

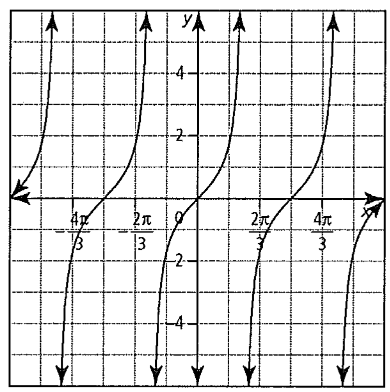
7. A small plane is flying at a constant altitude of 3000 m directly toward an observer. Assume the land in the area close to the observer is flat.

a) Draw a diagram to model the situation. Label the horizontal distance between the plane & the observer d, & the angle of elevation from the observer to the plane θ.

b) Write an equation that relates the distance to the angle of elevation.

c) At what angle is the plane directly above the observer? What is the distance, d, when the plane is directly above the observer?

8. Consider the graph.



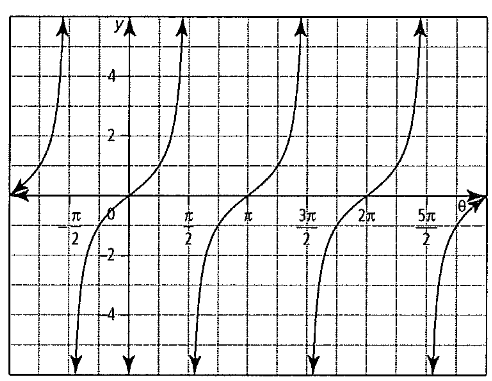
a) State the zeros of this function.

b) Where do the asymptotes of the function occur?

c) What is the domain of this function?

d) What is the range of this function?

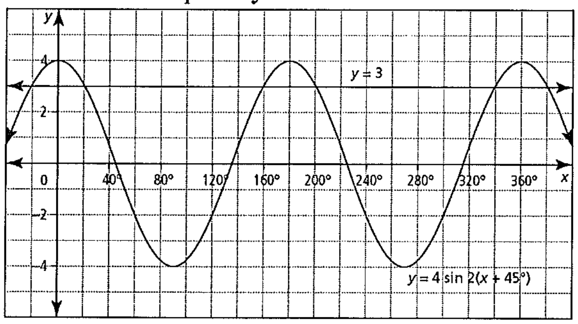
9. Use the graph of the function $y = \tan \theta$ to determine each value.



a) $\tan \pi$ **b)** $\tan\left(-\frac{\pi}{4}\right)$ **c)** $\tan 9\frac{\pi}{4}$ **d)** $\tan 5\frac{\pi}{2}$

Section 5.4 Extra Practice

1. The partial graphs of the functions $y = 4\sin 2(x + 45°)$ and the line $y = 3$ are shown. Determine the solutions to the equation $4\sin 2(x + 45°) = 3$ over the interval $0° \le x \le 360°$. Express your answers to the nearest degree.



2. For each situation, state a possible domain and range. & the period of each function to the nearest tenth of a unit.
   a) The motion of a point on an industrial flywheel can be described by the formula $h(t) = 13\cos\frac{2\pi}{0.7}t + 15$, where h is height, in metres, and t is the time, in seconds.
   b) The fox population in a particular region can be modelled by the equation $F(t) = 500\sin\frac{\pi}{12}t + 1000$, where F is the fox population and t is the time, in months.

3. In a 365-day year, a sinusoidal equation of the form $f(x) = a\cos b(x - c) + d$ can be used to graphically model the time of sunrise or sunset throughout the year, where $f(x)$ is the time of the day in decimal time format, and x is the day of the year. The sunrise and sunset times for Yellowknife are provided in the table.

	June 21 (172nd day of the year)	Dec 21 (355th day of the year)
Sunrise	2:34 a.m.	10:11 a.m.
Sunset	10:45 p.m.	3:00 p.m.

   a) Write an equation that models the time of sunrise in Yellowknife.
   b) Write an equation that models the time of sunset in Yellowknife.

4. At the bottom of its rotation, the tip of the blade on a windmill is 8 m above the ground. At the top of its rotation, the blade tip is 22 m above the ground. The blade rotates once every 5 s.
   a) Draw one complete cycle of this scenario.
   b) A bug is perched on the tip of the blade when the tip is at its lowest point. Determine the cosine equation of the graph for the bug's height over time.
   c) What is the bug's height after 4 s?
   d) How long is the bug more than 17m above ground?

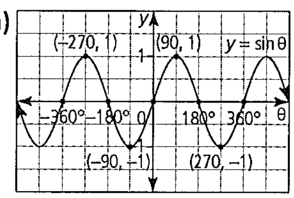
5. The average daily maximum temperature in Edmonton follows a sinusoidal pattern over the course of a year (365 days). Edmonton's highest temperature occurs on the 201st day of the year (July 20th) with an average high of 24 °C. Its coldest average temperature is −16 °C, occurring on January 14.
   a) Write a cosine equation for Edmonton's temperature over the course of the year.
   b) What is the expected average temperature for August 4th?
   c) For how many days is the average temperature higher than 20 °C?

6. The pendulum of a grandfather clock swings with a periodic motion that can be represented by a trigonometric function. At rest, the pendulum is 16 cm above the base. The highest point of the swing is 20 cm above the base, and it takes 2 s for the pendulum to swing back and forth once. Assume that the pendulum is released from its highest point.
   a) Write a cosine equation that models the height of the pendulum as a function of time.
   b) Write a sine equation that models the height of the pendulum as a function of time.

Answers

Section 5.1 Extra Practice

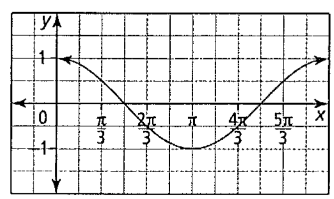
1. a)



b) $y = -\frac{\sqrt{2}}{2}$

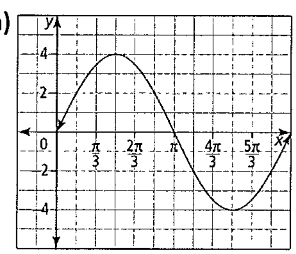
c) (−360, 0), (−180, 0), (0, 0), (180, 0), (360, 0)

2. a)



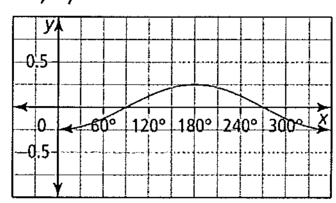
b) $y = -\frac{\sqrt{3}}{2}$ **c)** $y = -1$ **d)** (0, 1)

3. a)



b) $\{y \mid -4 \le y \le 4, y \in R\}$ **c)** 2π **d)** 4

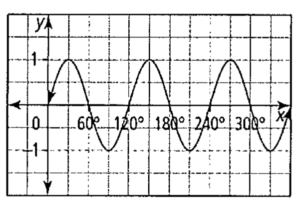
4a)



b) $(0, -\frac{1}{4})$ **c)** $\{y \mid -\frac{1}{4} \le y \le \frac{1}{4}, y \in R\}$

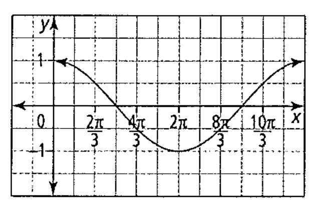
d) $\frac{1}{4}$

5. a)



b) 120° **c)** $\{y \mid -1 \le y \le 1, y \in R\}$ **d)** 1

6. a)



b) (0, 1) **c)** 4π **d)** 1

7. a) amp = 4, per = 180° or π **b)** amp = 3, per= 1800° or 10π
c) amp = $\frac{2}{3}$, per = 540° or 3π **d)** amp = $\frac{1}{4}$, per = 120° or $\frac{2\pi}{3}$

8. a) vert. exp. by a factor of 2, hor. compression by a factor of $\frac{1}{4}$

b) vert. refl. over the x-axis, horizontal expansion by a factor of 5

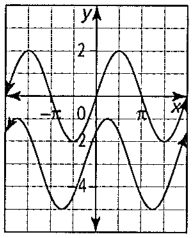
c) vertical reflection over the x-axis, vertical expansion by a factor of 3, horizontal compression by a factor of $\frac{2}{5}$

d) vert. exp. by a factor of 5, horizontal reflection over the y-axis

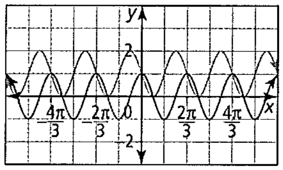
9. a) amp $= \frac{3}{4}$, per $= \frac{\pi}{3}$ or 60° **b)** amp = 2, per $= \frac{3\pi}{2}$ or 270°

Section 5.2 Extra Practice

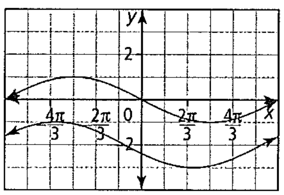
1. a)



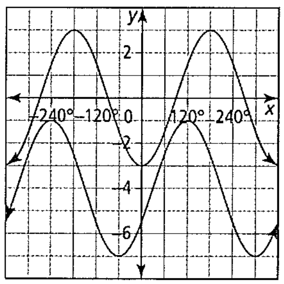
b)



c)



d)



2. a) phase shift = 25, vertical displacement = 3.2

b) phase shift $= \frac{-\pi}{6}$, vertical displacement = −7

c) phase shift $= \frac{\pi}{8}$, vertical displacement = 5

d) phase shift $= \frac{-2\pi}{3}$, vertical displacement = −1

3. a) period = 180°, range $= \{y \mid -10 \le y \le -2, y \in R\}$

b) period = 6π, range $= \{y \mid -1 \le y \le 5, y \in R\}$

c) period = 72°, range $= \{y \mid 1.9 \le y \le 6.5, y \in R\}$

d) period $= \frac{2\pi}{3}$, range $= \{y \mid -10 \le y \le 4, y \in R\}$

4. period $= \frac{2\pi}{|b|}$, range $= \{y \mid d - |a| \le y \le d + |a|, y \in R\}$

5. a) $y = 3 \sin 4\left(x - \frac{\pi}{2}\right) + 5$ **b)** $y = \frac{1}{2} \sin 3(x + 50°) - 4$

c) $y = \sin \frac{1}{4}\left(x - \frac{\pi}{2}\right)$ **d)** $y = \sin \frac{2}{3}x + 2$

6. Example: $y = 3 \sin 2\left(x + \frac{\pi}{4}\right)$

7. a) 5 **b)** −4 **c)** $\frac{2\pi}{3}$ **d)** $y = 5 \cos 3x - 4$

e) $y = 1$ for $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$ **f)** $y = -9$ for $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

8. Example: $y = 3 \sin 6(x - 105°) + 7$

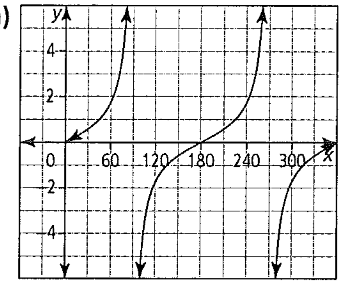
Section 5.3 Extra Practice

1. a) $\theta = 0, \theta = \pi, \theta = 2\pi$ **b)** $\theta = \frac{\pi}{4}, \theta = \frac{5\pi}{4}$

c) $\theta = \frac{3\pi}{4}, \theta = \frac{7\pi}{4}$ **d)** $\theta = \frac{\pi}{2}, \theta = \frac{3\pi}{2}$

2. a) $\frac{1}{\sqrt{3}}$ **b)** 1 **c)** $\sqrt{3}$ **d)** undefined **e)** $-\sqrt{3}$ **f)** −1 **g)** $\frac{-1}{\sqrt{3}}$ **h)** 0

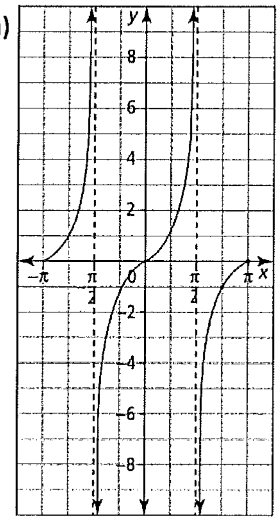
3. a)



b) $\{x \mid 0° \le x \le 360°, x \in R, x \ne 90°$ or $270°\}$

c) $\{y \mid y \in R\}$ **d)** 180°

4. a)



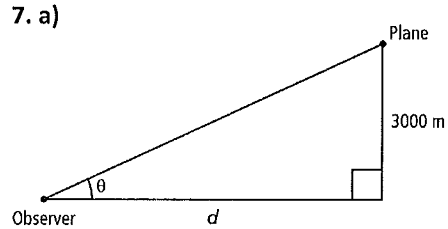
b) $(-\pi, 0), (0, 0), (\pi, 0)$

c) $x = \pm\frac{\pi}{2}$ **d)** 0

5. No, because it does not have maximum and minimum values.

6. asymptotes: $x = 90° + 180°n, n \in I$; domain: $\{x \mid x \ne 90° + 180°n, x \in R, n \in I\}$

7. a)



7b) $d = \frac{3000}{\tan\theta}$ **c)** $\theta = 90°, d = 0$ **8. a)** $x = n\pi, n \in I$ **b)** at $x = \frac{\pi}{2} + n\pi, n \in I$

c) $\{x \mid x \ne \frac{\pi}{2} + n\pi, x \in R, n \in I\}$ **d)** $\{y \mid y \in R\}$

9. a) 0 **b)** −1 **c)** 1 **d)** undefined

Section 5.4 Extra Practice

1. $x = 21°, 159°, 201°$, and $339°$

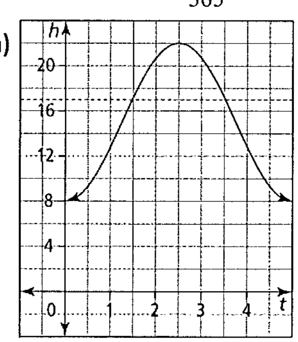
2. a) domain: $\{t \mid t \ge 0, t \in R\}$ range: $\{h \mid 2 \le h \le 28, h \in R\}$ period: 0.7m

b) domain: $\{t \mid t \ge 0, t \in R\}$ range: $\{F \mid 500 \le F \le 1500, F \in R\}$ period: 24 foxes

3. a) $T(x) = 3.808 \cos \frac{2\pi}{365}(x + 10) + 6.375$

b) $T(x) = -3.875 \cos \frac{2\pi}{365}(x + 10) + 18.875$

4. a)



b) $b(t) = -7 \cos \frac{2\pi}{5}t + 15$

c) $b(4) = 12.8$ m

d) $3.52 - 1.48 = 2.04$ s

5. a) $T(d) = 20 \cos \frac{2\pi}{365}(d - 201) + 4$

b) 23.3 °C **c)** 76 days

6. a) $h(t) = 2 \cos \pi t + 18$

b) $h(t) = 2 \sin \pi(t - 1.5) + 18$ or $h(t) = -2 \sin \pi(t - 0.5) + 18$