MA 12 LG 12 Review Sheet (Trigonometric Equations2)

1. Verify the possibility of an identity graphically then prove each identity algebraically.

a. $\text{Csc}^2x(1 - \text{Cos}^2x) = 1$

b. $\dfrac{\text{Sin}^2x + \text{Cos}^2x}{\text{Sec}x} = \text{Cos}x$

2. Prove each of the following identities and state any restrictions.

a. $\text{Sec}^4A - \text{Sec}^2A = \text{Tan}^4A + \text{Tan}^2A$

b. $\dfrac{\text{Sin}A + \text{Tan}A}{\text{Cos}A + 1} = \text{Tan}A$

3. Determine all restrictions:

a. $\dfrac{\text{Cot}A}{1 + \text{Sin}A}$

b. $\dfrac{1}{2\text{Cos}^2A + \text{Cos}A - 1}$

4. Simplify the folowing trig. expressions:

a. $\dfrac{\text{Csc}x\text{Cos}x}{\text{Tan}x}$ b. $\text{Sec}^2x - \text{Tan}^2x$

c. $\dfrac{1 + \text{Tan}x}{1 + \text{Cot}x}$ d. $\dfrac{\text{Sec}x}{\text{Sin}x} - \dfrac{\text{Sin}x}{\text{Cos}x}$

5. Give the exact value of each expression:

a. $\text{Sin}\frac{\pi}{3}\text{Cos}\frac{\pi}{4} - \text{Cos}\frac{\pi}{3}\text{Sin}\frac{\pi}{4}$

b. $\text{Cos}80°\text{Cos}50° + \text{Sin}80°\text{Sin}50°$

6. Find each of the following exactly using the appropriate sum or difference identities.

a. $\text{Cos}105°$ b. $\text{Sin}\frac{7\pi}{12}$

7. Express as a single trigonometric function.

a. $\text{Cos}^2\frac{\pi}{4} - \text{Sin}^2\frac{\pi}{4}$ b. $2\text{Sin}75°\text{Cos}75°$

c. $2\text{Cos}^2\frac{\pi}{3} - 1$

8. Find the exact value of each expression.

a. $2\text{Sin}67.5°\text{Cos}67.5°$

b. $3\text{Cos}^2\frac{\pi}{8} - 3\text{Sin}^2\frac{\pi}{8}$

9. Prove:

a. $\dfrac{1 + \text{Cos}2A}{\text{Sin}2A} = \text{Cot}A$

b. $\text{Cos}(90° - A) = \text{Sin}A$

10. Use the sum identity for sine to prove the double-angle identity for sine.

11. True or False?

a. $\text{Cos}28° = 1 - 2\text{Cos}^2 14°$

b. $\text{Sin}\frac{\pi}{6} = 2\text{Sin}3\pi\text{Cos}3\pi$

12. If A and B are in the second quadrant and $\text{Sin}A = \frac{3}{5}$ and $\text{Cos}B = \frac{-5}{13}$ find each of the following exactly:

a. $\text{Sin}(A + B)$ b. $\text{Cos}2B$

13. Prove each identity.

a. $\text{Sin}A(1 + \text{Csc}A) = 1 + \text{Sin}A$

b. $\dfrac{1}{1 - \text{Sin}A} + \dfrac{1}{1 + \text{Sin}A} = 2\text{Sec}^2A$

14. If $\text{Sin}B = \frac{4}{5}$, $\text{Cos}A = \frac{5}{13}$ and A and B are in Quadrant I, simplify $\text{Sin}(A + B)$.

15. Prove $\text{Sin}(A + \frac{\pi}{2}) = \text{Cos}A$.

16. Prove $\text{Sin}2A = 2\text{Cot}A\text{Sin}^2A$.

17. Evaluate $\text{Cos}2A$ if $\text{Sin}A = \frac{-2}{3}$ and A is Quad IV.

18. If $\text{Cos}A = \frac{-12}{13}$, $\pi < A < \frac{3\pi}{2}$, what is the exact value of $\text{Sin}2A$?

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Answer Key

1. a. $\text{Csc}^2x(1 - \text{Cos}^2x) = 1$

$\frac{1}{s^2}(s^2)$	1
1	1

b. $\text{SinxSecxCotx} = 1$

$\frac{1}{\frac{1}{c}}$	c
c	c

2. a. $\text{Sec}^4A - \text{Sec}^2A = \text{Tan}^4A + \text{Tan}^2A$

$\frac{1}{c^2}(\frac{1}{c^2} - 1)$	$\frac{s^2}{c^2}(\frac{s^2}{c^2} + 1)$
$\frac{1}{c^2}(\frac{1-c^2}{c^2})$	$\frac{s^2}{c^2}(\frac{s^2+c^2}{c^2})$
$\frac{1}{c^2}(\frac{s^2}{c^2})$	$\frac{s^2}{c^2}(\frac{1}{c^2})$

CosA≠0

b. $\frac{\text{SinA+TanA}}{\text{CosA+1}} = \text{TanA}$

$\frac{s + \frac{s}{c}}{c + 1}$	$\frac{s}{c}$
$\frac{cs + s}{c^2 + c}$	$\frac{s}{c}$
$\frac{s(c + 1)}{c(c + 1)}$	$\frac{s}{c}$
$\frac{s}{c}$	$\frac{s}{c}$

CosA≠0, CosA≠-1

3. a. SinA ≠ -1, SinA≠0
   b. $\text{CosA}\neq\frac{1}{2}$, CosA≠-1

4. a. Cot^2x b. 1 c. Tanx d. Cotx

5. a. $\frac{\sqrt{3}-1}{2\sqrt{2}}$ b. $\frac{\sqrt{3}}{2}$

6. a. $\text{Cos}(60^\circ + 45^\circ) = \frac{1-\sqrt{3}}{2\sqrt{2}}$
   b. $\text{Sin}(\frac{\pi}{4} + \frac{\pi}{3}) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

7. a. $\text{Cos}\frac{\pi}{2}$ b. Sin150°
   c. $\text{Cos}\frac{2\pi}{3}$

8. a. $\frac{1}{\sqrt{2}}$ b. $\frac{3}{\sqrt{2}}$

Answer Key

9. a. $\frac{\text{1+Cos2A}}{\text{Sin2a}} = \text{CotA}$

$\frac{1+2c^2-1}{2sc}$	$\frac{c}{s}$
$\frac{2c^2}{2sc}$	$\frac{c}{s}$
$\frac{c}{s}$	$\frac{c}{s}$

b. $\text{Cos}(90^\circ - A) = \text{SinA}$

Cos90Cos+ Sin90SinA	SinA
(0)Cos+ (1)SinA	SinA
SinA	SinA

10. a. $\text{Sin2A} = \text{2SinACosA}$

Sin(A+A)	2sc
SinACosA+ CosASinA	2sc
2sc	2sc

11. a. False b. False

12. a. $\frac{-63}{65}$ b. $\frac{-119}{169}$

13. a. $\text{SinA(1+CscA)} = \text{1+SinA}$

$s(1 + \frac{1}{s})$	1 + s
s + 1	1 + s
2sc	2sc

b. $\frac{1}{1-\text{SinA}} + \frac{1}{1+\text{SinA}} = \text{2Sec}^2\text{A}$

$\frac{1}{1-s} + \frac{1}{1+s}$	$\frac{2}{c^2}$
$\frac{1+s\ +\ 1-s}{(1-s)(1+s)}$	$\frac{2}{c^2}$
$\frac{2}{1-s^2}$	$\frac{2}{c^2}$
$\frac{2}{c^2}$	$\frac{2}{c^2}$

14. a. $\frac{56}{65}$

15. $\text{Sin}(A+\frac{\pi}{2}) = \text{CosA}$

$\text{SinACos}\frac{\pi}{2} + \text{CosASin}\frac{\pi}{2}$	c
SinA(0) + CosA(1)	c
c	c

16. $\text{Sin2A} = \text{2CotASin}^2\text{A}$

2sc	$2\frac{c}{s}(s^2)$
2sc	2sc

17. a. $\frac{1}{9}$

18. $\frac{120}{169}$