

6.1 Trig Identities

* All given on formula sheet

IB DP TIB 2014-05-26

$$\textcircled{1} \csc x = \frac{1}{\sin x}$$

$$\textcircled{2} \sec x = \frac{1}{\cos x}$$

$$\textcircled{3} \cot x = \frac{1}{\tan x}$$

$$\textcircled{4} \tan x = \frac{\sin x}{\cos x} \quad \tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

$$\textcircled{5} \cot x = \frac{\cos x}{\sin x}$$

$$\textcircled{6} \sin^2 x + \cos^2 x = 1$$

$\cos^2 x = 1 - \sin^2 x$

$$\textcircled{7} 1 + \tan^2 x = \sec^2 x$$

$$\textcircled{8} 1 + \cot^2 x = \csc^2 x$$

$$\textcircled{2} \sin x + \cos x \cot x$$

$$\sin x + \cos x \left(\frac{\cos x}{\sin x} \right)$$

$$\frac{(\sin x) \sin x + \cos^2 x}{(\sin x)}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$$

Simplify

* Often more than one way / Use formula sheet

$$\textcircled{1} \frac{2 \cos x}{1 - \sin^2 x}$$

$$\frac{2 \cos x}{\cos^2 x} = \frac{2}{\cos x} = 2 \left(\frac{1}{\cos x} \right) = 2 \sec x$$

$$\textcircled{3} (\sec^2 x - 1) (\cot^2 x)$$

$$\left(\tan^2 x \right) \left(\frac{1}{\tan^2 x} \right) \text{ OR } (\tan^2 x) \left(\frac{\cos^2 x}{\sin^2 x} \right)$$

$$1$$

$$\left(\frac{\sin^2 x}{\cos^2 x} \right) \left(\frac{\cos^2 x}{\sin^2 x} \right)$$

$$1$$

4) $\frac{\cos x \cot x + \cos x}{\cot x + \cot^3 x}$

* Hint: Factor first

$$\frac{\cos x (\cot x + 1)}{\cot x (1 + \cot x)}$$

$$\frac{\cos x}{\cot x} = \frac{\cos x \cdot \sin x}{\frac{\cos x}{\sin x}} = \boxed{\sin x}$$

5) $\frac{\sin x}{1 + \cos x} + \frac{\cos x}{\sin x}$ * Common Denominator

$$\frac{\sin x (\sin x)}{(1 + \cos x)(\sin x)} + \frac{\cos x (1 + \cos x)}{\sin x (1 + \cos x)}$$

$$= \frac{\sin^2 x + \cos x + \cos^3 x}{(1 + \cos x)(\sin x)}$$

$$= \frac{1 + \cos x - 1}{(1 + \cos x)(\sin x)} = \boxed{\csc x}$$

6) $\frac{1 + \sin x}{\cos x} - \frac{\cos x}{1 - \sin x}$

Pg 264

* | (all)

$$\frac{1 + \sin x (1 - \sin x)}{\cos x (1 - \sin x)} - \frac{\cos x (\cos x)}{1 - \sin x (\cos x)} \quad \begin{matrix} 2(a,b,c), 3(c,e,i) \\ 7(all) \end{matrix}$$

$$\frac{1 - \sin^2 x}{\cos x (1 - \sin x)} - \frac{\cos^2 x}{\cos x (1 - \sin x)} \quad \text{OR} \quad \frac{1 - \sin^2 x - \cos^2 x}{\cos x (1 - \sin x)}$$

$$\frac{\cos^2 x - \cos^2 x}{\cos x (1 - \sin x)} = \boxed{0}$$

$$\frac{1 - (\sin^2 x + \cos^2 x)}{\cos x (1 - \sin x)}$$

$$\frac{1 - 1}{\cos x (1 - \sin x)} = \boxed{0}$$

6.1 Tris Identities Pt. 2

Made This

2011-02-06

Simplify

$$\textcircled{1} \frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x}$$

$$\frac{\sin x (1 - \cos x)}{1 - \cos^2 x} + \frac{\sin x (1 + \cos x)}{1 - \cos^2 x}$$

$$\frac{\sin x - \sin x \cos x + \sin x + \sin x \cos x}{(1 - \cos^2 x)(1 - \cos x)}$$

$$\frac{2 \sin x}{1 - \cos^2 x} = \frac{2 \sin x}{\sin^2 x} = 2 \left(\frac{1}{\sin x} \right) = 2 \csc x$$

$$\textcircled{2} \frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1}$$

$$\frac{\cos x (\sec x + 1)}{(\sec x - 1)(\sec x + 1)} + \frac{\cos x (\sec x - 1)}{(\sec x + 1)(\sec x - 1)}$$

$$\frac{1 + \cos x + 1 - \cos x}{\sec^2 x - 1}$$

$$= \frac{2}{\tan^2 x} = 2 \cot^2 x$$

Simplify & state the restrictions for $0 < \theta < 2\pi$

$$\textcircled{3} \sin \theta \sec \theta$$

$$\sin \theta \left(\frac{1}{\cos \theta} \right)$$

$$\frac{\sin \theta}{\cos \theta}$$

$$\tan \theta$$

Restrictions: $\cos \theta \neq 0$



$$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

Simplify & state the restrictions for $0 < x < 2\pi$

$$\textcircled{4} \frac{\sin x + \tan x}{\cos x + 1}$$

$$\frac{\sin x + \frac{\sin x}{\cos x}}{\cos x + 1}$$

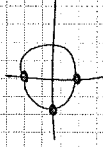
$$\frac{\sin x \cos x + \sin x}{\cos x (\cos x + 1)} = \frac{\sin x (\cos x + 1)}{\cos x (\cos x + 1)}$$

$$\cos x + 1$$

$$= \frac{\sin x}{\cos x} = \tan x$$

$$\cos x \neq -1$$

$$\cos x \neq 0$$



$$x \neq \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

$$\textcircled{5} \frac{1 + \tan x}{1 + \cot x}$$

$$\begin{aligned} \frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}} &= \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}} \\ &= \frac{\cos x + \sin x}{\cos x} \cdot \frac{\sin x}{\sin x + \cos x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \end{aligned}$$

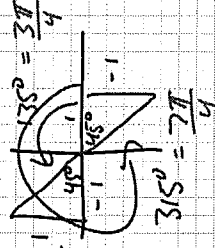
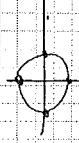
$$\frac{1 + \tan x}{1 + \cot x}$$

$$\begin{aligned} \frac{1 + \tan x}{1 + \frac{1}{\tan x}} &= \frac{1 + \tan x}{\frac{\tan x + 1}{\tan x}} \\ &= \frac{1 + \tan x}{1 + \tan x} \cdot \frac{\tan x}{\tan x} \\ &= \tan x \end{aligned}$$

$$\cos x \neq 0$$

$$\sin x \neq 0$$

$$\cot x \neq -1$$



$$x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{7\pi}{4}$$

$$P_2 264 \# 2(c, d, f) \# 3(f, g), 6$$

*WORKSHEET

6.2 Verifying Trig Identities Pt. 1

MEMO TIP: 2015-11-27

HINTS / STRATEGIES FOR PROVING IDENTITIES

1. Convert both sides to sines & cosines
2. Start with more complicated side
3. If you have a binomial, multiply the top and bottom by the conjugate.
 Ex $\frac{\sin x (1 + \cos x)}{1 - \cos x (1 + \cos x)}$ OR $\frac{1 + \sin x (1 - \sin x)}{\cos x (1 - \sin x)}$
4. Force the denominator
 Ex $\frac{\csc x}{\tan x + \cot x} = \csc x \frac{(1 + \sin x)(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$

① Prove $1 + \tan^2 x = \sec^2 x$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x}$$

$$\sec^2 x$$

L.S. = R.S.

① Prove $\frac{\cos \theta}{1 + \sin \theta} + \frac{\cos \theta}{1 - \sin \theta} = 2 \sec \theta$

$$\frac{\cos \theta (1 - \sin \theta)}{1 + \sin \theta (1 - \sin \theta)} + \frac{\cos \theta (1 + \sin \theta)}{1 - \sin \theta (1 + \sin \theta)}$$

$$\frac{\cos \theta - \cos \theta \sin \theta}{1 - \sin^2 \theta} + \frac{\cos \theta + \cos \theta \sin \theta}{1 - \sin^2 \theta}$$

$$\frac{2 \cos \theta}{\cos^2 \theta} = \frac{2}{\cos \theta} = 2 \left(\frac{1}{\cos \theta} \right)$$

$$2 \sec \theta$$

L.S. = R.S.

② Prove $\frac{\cos \theta}{\sec \theta - 1} + \frac{\cos \theta}{\sec \theta + 1} = 2 \cot^2 \theta$

$$\frac{\cos \theta (\sec \theta + 1)}{\sec \theta - 1 (\sec \theta + 1)} + \frac{\cos \theta (\sec \theta - 1)}{\sec \theta + 1 (\sec \theta - 1)}$$

$$\frac{\cos \theta \sec \theta + \cos \theta}{\sec^2 \theta - 1} + \frac{\cos \theta \sec \theta - \cos \theta}{\sec^2 \theta - 1}$$

$$\frac{2 \cos \theta \sec \theta}{\sec^2 \theta - 1} = \frac{2}{\tan^2 \theta}$$

$$= 2 \cot^2 \theta$$

L.S. = R.S.

3) Prove $\frac{\sec x}{\cot x + \tan x} = \sin x$

$$\frac{\sec x}{\cot x + \tan x} = \frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$$

$$= \frac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}}$$

$$= \frac{1}{\cos x} \cdot \frac{\sin x \cos x}{\cos^2 x + \sin^2 x}$$

$$= \frac{\sin x}{\cos^2 x + \sin^2 x}$$

$$= \frac{\sin x}{1} = \sin x$$

L.S. = R.S.

4) Prove $\csc x + \cot x = \frac{\sin x}{1 - \cos x}$

$$\frac{1}{\sin x} + \frac{\cos x}{\sin x} = \frac{1 + \cos x}{\sin x}$$

$$= \frac{(1 + \cos x)(1 - \cos x)}{\sin x (1 - \cos x)}$$

$$= \frac{1 - \cos^2 x}{\sin x (1 - \cos x)}$$

$$= \frac{\sin^2 x}{\sin x (1 - \cos x)}$$

$$= \frac{\sin x}{1 - \cos x}$$

L.S. = R.S.

State the restriction $0 \leq x < 2\pi$

$\sin x \neq 0, \cos x \neq 1$
 $x \neq 0, \pi$

pg 271
 * 1-25
 (odds)

6.2 Pt. 2

Neo 11/2

2015-11-21

PROVE + STATE THE RESTRICTIONS $0 < \theta < 2\pi$

$$\textcircled{1} \frac{\sec \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \cot \theta$$

$$\textcircled{1} \frac{\sec \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \cot \theta$$

$$\frac{1}{\sin \theta \cos \theta} - \frac{\sin \theta (\sin \theta)}{\cos \theta (\sin \theta)}$$

$$\frac{1 - \sin^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos \theta}{\sin \theta}$$

$$\cot \theta$$

L.S. = R.S.

Restrictions:

$$\sin \theta \neq 0 \quad \cos \theta \neq 0$$



$$\theta \neq 0, \pi, \pi/2, 3\pi/2$$

$$\textcircled{2} \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

$$\textcircled{2} \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$$

$$\frac{\sin \theta (1 - \cos \theta)}{1 + \cos \theta (1 - \cos \theta)}$$

$$\frac{\sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta}$$

$$\frac{\sin \theta (1 - \cos \theta)}{\sin^2 \theta}$$

$$\frac{1 - \cos \theta}{\sin \theta}$$

L.S. = R.S.

Restrictions:

$$\cos \theta \neq \pm 1 \quad \sin \theta \neq 0$$



$$\theta \neq 0, \pi$$

$$\textcircled{3} \frac{1 - \cos \theta}{\sin \theta} = \frac{1}{\csc \theta + \cot \theta}$$

$$\textcircled{3} \frac{1 - \cos \theta}{\sin \theta} = \frac{1}{\csc \theta + \cot \theta}$$

$$\frac{1 + \cos \theta}{\sin \theta} = \frac{1}{\csc \theta - \cot \theta}$$

$$\frac{1 + \cos \theta}{\sin \theta} = \frac{1}{\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}}$$

$$\frac{1 + \cos \theta}{\sin \theta} = \frac{1}{\frac{1 - \cos \theta}{\sin \theta}}$$

$$\frac{1 + \cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{1 - \cos \theta} = \frac{1}{1}$$

$$\frac{1 + \cos \theta}{1 - \cos \theta} = \frac{1}{1}$$

$$\frac{1 + \cos \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{1 - \cos \theta} \cdot \frac{1 + \cos \theta}{1 + \cos \theta} = \frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta} = \frac{(1 + \cos \theta)^2}{\sin^2 \theta}$$

$$\frac{1 + \cos \theta}{1 - \cos \theta} = \frac{(1 + \cos \theta)^2}{\sin^2 \theta}$$

$$\frac{1}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin^2 \theta}$$

$$\frac{1}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin^2 \theta}$$

$$\frac{1}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin^2 \theta}$$

Restrictions

$$\sin \theta \neq 0 \quad \cos \theta \neq \pm 1$$



$$\theta \neq 0, \pi$$

L.S. = R.S.

$$\textcircled{4} (\tan x - 1)^2 = \sec^2 x - 2 \tan x$$

$$\textcircled{4} (\tan x - 1)^2 = \sec^2 x - 2 \tan x$$

$$(\tan x - 1)(\tan x - 1)$$

$$\tan^2 x - 2 \tan x + 1$$

$$\tan^2 x + 1 - 2 \tan x$$

$$\sec^2 x - 2 \tan x$$

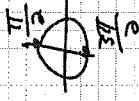
L.S. = R.S.

Restrictions

$$\cos x \neq 0$$

$$\tan x = \frac{\sin x}{\cos x} \rightarrow$$

$$\sec^2 x = \frac{1}{\cos^2 x}$$



$$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

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2-24
(evens)

6.3 Solving Trig Equations Pt. 1

PROB 718 2011-06-11

To solve a trig equation:

1) Factor if possible → "Determine exact values"

2) Calculator → $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$
→ "Decimal places"

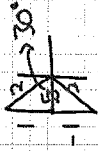
3) Quadratic Formula → Some trig functions but can't be factored

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

① Solve $0 < \theta < 2\pi$

a) $2 \cos \theta + \sqrt{3} = 0$ b) $2 \cos 3\theta + \sqrt{3} = 0$

$$\cos \theta = \frac{-\sqrt{3}}{2}$$



$$\theta = 150^\circ, 210^\circ$$

$$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$\cos 3\theta = \frac{-\sqrt{3}}{2} \quad \cos \theta = \frac{-\sqrt{3}}{2}?$$

$\frac{5\pi}{6} \times \frac{1}{3}$ $\frac{7\pi}{6} \times \frac{1}{3}$ Gives the first two answers

Add the period up to 2π

$$T = \frac{2\pi}{3} = \frac{12\pi}{18}$$

Start at 2π

$$\frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$$

d) $\sqrt{2} \cos 3\theta = 1$

$$\cos 3\theta = \frac{1}{\sqrt{2}}$$

$$\cos \theta = \frac{1}{\sqrt{2}}?$$



$$45^\circ, 315^\circ$$

$$\frac{\pi}{4}, \frac{7\pi}{4}$$

$$+ T = \frac{2\pi}{3} = \frac{8\pi}{12}$$

$$\frac{\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$$

c) $2 \sin 4\theta = -1$

$$\sin 4\theta = -\frac{1}{2}$$

$$\sin \theta = -\frac{1}{2}?$$



$$210^\circ, 330^\circ$$

$$\frac{7\pi}{6}, \frac{11\pi}{6}$$

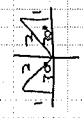
$$+ T = \frac{12\pi}{24}$$

$$\frac{7\pi}{24}, \frac{11\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}, \frac{31\pi}{24}, \frac{35\pi}{24}, \frac{43\pi}{24}, \frac{47\pi}{24}$$

e) $2 \sin \frac{1}{3} x = 1$

$\sin \frac{1}{3} x = \frac{1}{2}$

$\sin x = \frac{1}{2} ?$

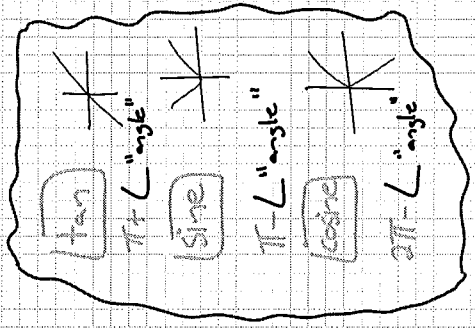


$x = 30^\circ, 150^\circ$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\times 2 \quad x = \frac{2\pi}{6}, \frac{10\pi}{6}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$



When using calculator:

② Solve $0 < x < 2\pi$

a) $\sec x = -2.3124$

$\cos x = \frac{1}{-2.3124}$

$x = \cos^{-1} \left(-\frac{1}{2.3124} \right)$

$x = 2.02 \quad \cos: 2\pi -$

$2\pi - 2.02$
 4.27

$x = 2.02, 4.27$

b) $\sec 2x = -2.3124$

$x \frac{1}{2}$ and add period $\frac{2\pi}{2} = \pi$

$\frac{2.02}{2} = 1.01 \quad \frac{4.27}{2} = 2.13$

$x = 1.01, 2.13, 4.15, 5.27$

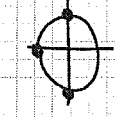
$+ \pi$

③ Determine the exact values of $\sin^2 x - \sin x = 0$,

where $0^\circ \leq x < 360^\circ$.

$\sin x (\sin x - 1) = 0$

$\sin x = 0 \quad \sin x = 1$



$x = 0^\circ, 90^\circ, 180^\circ$

4) Find the exact values for $2\cos^2 x - \cos x - 1 = 0$,
 where $0 \leq x < 2\pi$.

2018-04-15

Let $y = \cos x$ "Subst Variable"

$$2y^2 - y - 1 = 0 \quad \rightarrow \quad 2y(y-1) + 1(y-1) = 0$$

$$(2y+1)(y-1) = 0$$

$$y = -\frac{1}{2} \quad y = 1$$

$$2y^2 - 2y + y - 1 = 0$$

$$\therefore \cos x = -\frac{1}{2} \quad \cos x = 1$$

$$\cos x = -\frac{1}{2}$$



$$x = 120^\circ, 240^\circ$$

$$\frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\cos x = 1$$



$$x = 0^\circ$$

$$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

5) Solve $6\sin^2 x + 11\sin x - 10 = 0$, $0 \leq x < 2\pi$

Let $y = \sin x$

$$6y^2 + 11y - 10 = 0$$

$$6y^2 + 15y - 4y - 10 = 0$$

$$3y(2y+5) - 2(2y+5) = 0$$

$$(3y-2)(2y+5) = 0$$

$$y = \frac{2}{3} \quad y = -\frac{5}{2}$$

$H_{\text{yp}} > 0$ $H_{\text{yp}} > 0$
or Hyp

$$\sin x = \frac{2}{3}$$

$$\pi - 0.73$$

$$x = \sin^{-1}\left(\frac{2}{3}\right)$$

$$x = 0.73 \text{ AND } 2.41$$

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1-5 (a, c, e)

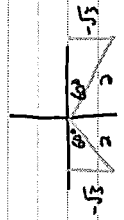
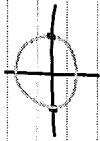
(ONLY $0 \leq x < 2\pi$)

6.3 Trig Equations Pt. 2

2008-01-18

① Solve $2\sin^2 x + \sqrt{3}\sin x = 0$, $0 \leq x < 2\pi$

$\sin x (2\sin x + \sqrt{3}) = 0$ $\sin x = 0$ $\sin x = -\frac{\sqrt{3}}{2}$

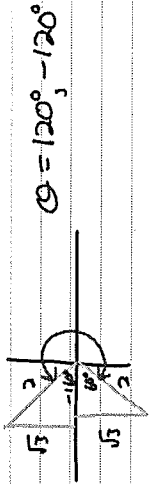


$x = 0, \pi$

$x = 240^\circ, 300^\circ$

$x = 0, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

② Solve $\cos \theta = -\frac{1}{2}$, $-\pi \leq \theta \leq \pi$



$\cos \theta = -\frac{1}{2}$

$\theta = 120^\circ, -120^\circ$

$\theta = \frac{2\pi}{3}, -\frac{2\pi}{3}$

③ Solve $\csc \theta = -2$ $-\pi \leq \theta \leq \pi$

$\sin \theta = \frac{1}{-2}$



$\theta = -30^\circ, -150^\circ$

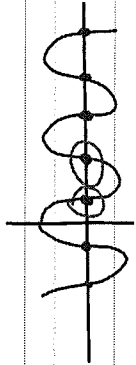
$\theta = -\frac{\pi}{6}, -\frac{5\pi}{6}$

GENERAL SOLUTION

To determine the general solution:

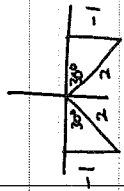
1. Find the first 2 answers $\rightarrow \sin/\cos \frac{2\pi}{6} + \tan \frac{\pi}{6}$
2. Add n times the period where $n \in \mathbb{I}$.

* Gives a summary of all of the possible solutions.



③ Determine the general solution for:

a) $\sin x = -\frac{1}{2}$



$x = 210^\circ, 330^\circ$ $T = 360^\circ$

OR

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $T = 2\pi$

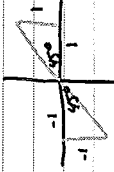
$x = 210^\circ + n360^\circ$ $n \in \mathbb{I}$ OR $x = \frac{7\pi}{6} + n2\pi$ $n \in \mathbb{I}$
 $x = 330^\circ + n360^\circ$ $n \in \mathbb{I}$ OR $x = \frac{11\pi}{6} + n2\pi$ $n \in \mathbb{I}$

b) $(\cos x + 1)(\tan x - 1) = 0$

$\cos x = -1$ $\tan x = 1$



$x = \pi$



$x = 45^\circ, 225^\circ$

OR
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \pi + n2\pi$ $n \in \mathbb{I}$
 $x = \frac{\pi}{4} + n\pi$
 $x = \frac{5\pi}{4} + n\pi$

④ Solve $\tan^2 x + 6 \tan x - 7 = 0$ over the real numbers.
(Same as general solution)

Let $y = \tan x$ $\tan x = 1$

$x = \tan^{-1}(1)$



$y^2 + 6y - 7 = 0$

$(y+7)(y-1) = 0$

$x = -1.43$

(Must be positive + π)

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$y = -7, y = 1$

$x = 1.71, 4.85$

$x = 1.71 + n\pi$ $n \in \mathbb{I}$
 $x = \frac{\pi}{4} + n\pi$

⑤ Solve $\cos^2 2x + \cos 2x = 0$:

a) $0 < x < 2\pi$

$\cos 2x (\cos 2x + 1) = 0$

$\cos 2x = 0$ $\cos 2x = -1$

$\cos x = 0$? $\cos x = -1$?



$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\cos 2x$:

Multiply by $\frac{1}{2}$ since $\cos 2x$ + add the period (π)

$$x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\begin{aligned} & \leftarrow + \frac{4\pi}{4} \\ & \leftarrow + \frac{2\pi}{2} \\ & \leftarrow + \frac{4\pi}{4} \end{aligned}$$

b) General Solution?

$$\frac{\pi}{4} + n\pi$$

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$$\frac{3\pi}{4} + n\pi$$

* 1-5 (b,d,f)

$$\frac{\pi}{2} + n\pi$$

↳ General Solution Only

$n \in \mathbb{Z}$

6.4 Sum and Difference Identities

NEP: TIPS

2012-12-01

$$\textcircled{1} \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\textcircled{2} \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\textcircled{3} \sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$\textcircled{4} \sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\textcircled{5} \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\textcircled{6} \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

① Express as a single trig function

$$\sin\left(\frac{5\pi}{6}\right) \cos\left(\frac{3\pi}{8}\right) - \sin\left(\frac{3\pi}{8}\right) \cos\left(\frac{5\pi}{6}\right)$$

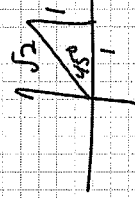
$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\sin\left(\frac{5\pi}{6} - \frac{3\pi}{8}\right)$$

$$\sin\left(\frac{20\pi}{24} - \frac{9\pi}{24}\right) = \sin\frac{11\pi}{24}$$

② Simplify

$$\begin{aligned} \text{a) } & \frac{\tan\frac{2\pi}{5} - \tan\frac{3\pi}{20}}{1 + \tan\frac{2\pi}{5} \tan\frac{3\pi}{20}} \\ &= \tan\left(\frac{2\pi}{5} - \frac{3\pi}{20}\right) \\ &= \tan\left(\frac{8\pi}{20} - \frac{3\pi}{20}\right) = \tan\left(\frac{5\pi}{20}\right) = \tan\frac{\pi}{4} = 1 \end{aligned}$$



↑
"single trig function"

$$\text{b) } \frac{\sin 5x}{\sec x} - \frac{\cos 5x}{\csc x}$$

$$\frac{\sin 5x}{\frac{1}{\cos x}} - \frac{\cos 5x}{\frac{1}{\sin x}}$$

$$\sin 5x \cos x - \sin x \cos 5x$$

$$\sin(5x - x)$$

$$\sin 4x$$

(2) Determine the exact value

a) $\cos 170^\circ \cos 50^\circ + \sin 170^\circ \sin 50^\circ$



$$\cos(170^\circ - 50^\circ) = \cos 120^\circ$$

$$= \boxed{-\frac{1}{2}}$$

b) $\sin 75^\circ$

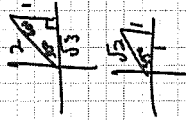
$$= \sin(30^\circ + 45^\circ)$$

$$= \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \boxed{\frac{1+\sqrt{3}}{2\sqrt{2}}}$$

$$\text{OR } \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$



c) $\sec 15^\circ = \frac{1}{\cos 15^\circ} = \frac{1}{\cos(45^\circ - 30^\circ)}$



$$\frac{1}{\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{1}{\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}}$$

$$= \frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

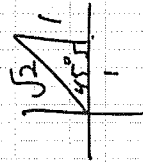
$$= \boxed{\frac{2\sqrt{2}}{\sqrt{3}+1}}$$

$$\text{OR } \frac{2\sqrt{2}(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

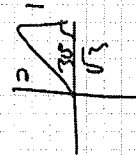
$$\boxed{\frac{\sqrt{6}-\sqrt{2}}{2}}$$

c) $\tan 15^\circ$

$$\tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$



$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$



$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

d) $\cos \frac{7\pi}{12}$

$$= \cos \left(\frac{4\pi}{12} + \frac{3\pi}{12} \right)$$

$$= \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4}$$

$$= \left(\frac{1}{2} \right) \left(\frac{1}{\sqrt{2}} \right) - \left(\frac{\sqrt{3}}{2} \right) \left(\frac{1}{\sqrt{2}} \right)$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

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* 1,2 (not i,j)

$$\text{OR } \frac{\sqrt{2} - \sqrt{6}}{4}$$



e) $\cot \left(-\frac{5\pi}{12} \right)$

$$= \cot \left(-\frac{2\pi}{12} - \frac{3\pi}{12} \right)$$

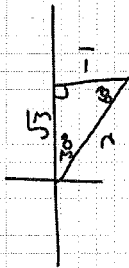
$$= \cot \left(-\frac{\pi}{6} - \frac{\pi}{4} \right)$$

$$= \frac{1}{\tan \left(-\frac{\pi}{6} - \frac{\pi}{4} \right)} = \frac{1}{\frac{\tan \left(-\frac{\pi}{6} \right) - \tan \left(\frac{\pi}{4} \right)}{1 + \tan \left(-\frac{\pi}{6} \right) \tan \left(\frac{\pi}{4} \right)}}$$

$$= \frac{-\frac{\pi}{6} - \frac{\pi}{4}}{1 - \frac{3\pi}{12} - \frac{\pi}{12}}$$

$$= \frac{-\frac{2\pi - 3\pi}{12} - \frac{\pi}{12}}{1 - \frac{4\pi}{12}}$$

$$= \frac{1}{\frac{\tan \left(-\frac{\pi}{6} \right) - \tan \left(\frac{\pi}{4} \right)}{1 + \tan \left(-\frac{\pi}{6} \right) \tan \left(\frac{\pi}{4} \right)}}$$



$$= \frac{1}{\frac{-\frac{1}{\sqrt{3}} - 1}{1 + \left(-\frac{1}{\sqrt{3}} \right) (1)}} = \frac{1 - \frac{1}{\sqrt{3}}}{-\frac{1}{\sqrt{3}} - 1}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} - 1 - \sqrt{3}} = \frac{\sqrt{3} - 1}{-1 - \sqrt{3}}$$

$$\text{OR } \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

$$\text{OR } \frac{1 - 2\sqrt{3} + 3}{-2} = \frac{4 - 2\sqrt{3}}{-2}$$

$$\boxed{\sqrt{3} - 2}$$

Rg 292

1, 2 (not i, j)

6.4 Pt. 1 Review

MSB 1109

2015-12-04

① Express as a single trig function & determine the exact value

a) $\cos 80^\circ \cos 50^\circ + \sin 80^\circ \sin 50^\circ$

$$\cos(80^\circ - 50^\circ) = \boxed{\cos 30^\circ}$$



b) $\sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4}$

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{4\pi - 3\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{12}\right)$$

$$\frac{\frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \left(\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right)}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

6.4 Pt. 2

MSB 1109

2015-12-03

① If both A and B are third quadrant angles, what is the value of $\sin(A-B)$ if $\sin A = -\frac{3}{5}$ and $\cos B = \frac{12}{13}$?

$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$= \left(-\frac{3}{5}\right) \left(\frac{12}{13}\right) - \sin B \cos A$$



$$\begin{aligned} x &= \sqrt{5^2 - (-3)^2} \\ x &= \sqrt{25 - 9} \quad \cos A = \frac{4}{5} \\ x &= \sqrt{16} = -4 \end{aligned}$$

$$y = \sqrt{13^2 - (12)^2}$$

$$\begin{aligned} y &= \sqrt{25} \quad \sin B = \frac{-5}{13} \\ y &= -5 \end{aligned}$$

$$= \left(-\frac{3}{5}\right) \left(\frac{12}{13}\right) - \sin B \cos A$$

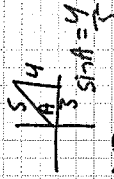
$$= \left(-\frac{3}{5}\right) \left(\frac{12}{13}\right) - \left(-\frac{5}{13}\right) \left(-\frac{4}{5}\right)$$

$$= \frac{36}{65} - \frac{20}{65}$$

$$= \boxed{\frac{16}{65}}$$

② If $\cos A = \frac{3}{5}$, $0 \leq A < \frac{\pi}{2}$ and $\sin B = \frac{12}{13}$, $\frac{\pi}{2} \leq B < \pi$

determine the exact value of $\cos(A+B)$.



$$\sin A = \frac{4}{5}$$



$$\cos B = \frac{-5}{13}$$

$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ &= \left(\frac{3}{5}\right) \cos B - \sin A \left(\frac{12}{13}\right) \end{aligned}$$

$$= \left(\frac{3}{5}\right) \left(\frac{-5}{13}\right) - \left(\frac{4}{5}\right) \left(\frac{12}{13}\right)$$

$$= \frac{-15}{65} - \frac{48}{65}$$

$$= \boxed{\frac{-63}{65}}$$

③ Simplify $\sin(x + \frac{\pi}{3}) + \sin(x - \frac{\pi}{3})$

$$\sin x \cos \frac{\pi}{3} + \cancel{\sin \frac{\pi}{3} \cos x} + \sin x \cos \frac{\pi}{3} - \cancel{\sin \frac{\pi}{3} \cos x}$$

$$= 2 \sin x \cos \frac{\pi}{3}$$

$$= 2 \sin x \left(\frac{1}{2}\right)$$

$$= \boxed{\sin x}$$

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→ 3 (b, c, e)

6 (s, c)

6.5 Double-Angle Identities

Math Tips

2015-12-04

$$① \sin 2\theta = 2 \sin \theta \cos \theta$$

$$② \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$1 - 2 \sin^2 \theta$$

$$2 \cos^2 \theta - 1$$

$$③ \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin 4\theta = \sin 2(2\theta)$$

$$= 2 \sin 2\theta \cos 2\theta$$

$$\sin 10x = \sin 2(5x)$$

$$= 2 \sin 5x \cos 5x$$

$$\sin \theta \cos \theta = \frac{\sin 2\theta}{2}$$



$$\sin 2\left(\frac{\pi}{6}\right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2}$$

① Simplify & determine the exact value

a) $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$

$$2 \sin \theta \cos \theta = \sin 2\theta$$

b) $\sin \frac{5\pi}{12} \cos \frac{5\pi}{12}$

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad \therefore \sin \theta \cos \theta = \frac{\sin 2\theta}{2}$$

$$\frac{\sin 2\left(\frac{5\pi}{12}\right)}{2} = \frac{\sin \left(\frac{5\pi}{6}\right)}{2}$$

$$= \frac{\frac{1}{2}}{2} = \frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$= \frac{1}{4}$$



c) $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$

$$\cos^2 \theta - \sin^2 \theta = \cos 2\theta$$

$$= \cos 2\left(\frac{\pi}{12}\right) = \cos \frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2}$$

$$d) 12 \cos^2 \frac{\pi}{3} - 6$$

$$6 \left(2 \cos^2 \frac{\pi}{3} - 1 \right)$$

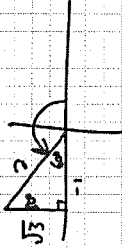
$$6 \left(\cos 2 \left(\frac{\pi}{3} \right) \right)$$

$$6 \left(\cos \frac{2\pi}{3} \right)$$

$$6 \left(-\frac{1}{2} \right)$$

$$\boxed{-3}$$

$$\rightarrow \cos 2\theta = 2 \cos^2 \theta - 1$$



$$3) \text{ Prove } \frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} = \csc \theta \text{ and state restrictions.}$$

$$\frac{\cos 2\theta \cos \theta}{\sin \theta \cos \theta} + \frac{\sin 2\theta \sin \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos 2\theta \cos \theta + \sin 2\theta \sin \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos(2\theta - \theta)}{\sin \theta \cos \theta}$$

$$\frac{\cos \theta}{\sin \theta \cos \theta} \frac{1}{\sin \theta} = \csc \theta \quad \text{L.S.} = \text{R.S.}$$

Restrictions:

$$\sin \theta \neq 0, \cos \theta \neq 0$$



$$\theta \neq n\pi/2, n \in \mathbb{I}$$

$$2) \text{ Prove } \frac{1}{1 - \sin \theta} = \frac{2 \sin \theta (\sec \theta + \tan \theta)}{\sin 2\theta}$$

$$\frac{2 \sin \theta \left(\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \right)}{2 \sin \theta \cos \theta}$$

$$\frac{\frac{1 + \sin \theta}{\cos \theta}}{\cos \theta} \quad \text{L.S.} = \text{R.S.}$$

$$\frac{1 + \sin \theta}{1 - \sin \theta}$$

$$\frac{1 + \sin \theta}{\cos^2 \theta}$$

$$\frac{1 + \sin \theta}{\cos^2 \theta} \rightarrow \text{OR } \frac{1 + \sin \theta}{1 - \sin \theta}$$

$$\frac{1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)}$$

$$\frac{1}{1 - \sin \theta}$$

$$\text{OR } \frac{\cos 2\theta}{\sin \theta} + \frac{\sin 2\theta}{\cos \theta} = \csc \theta$$

$$\frac{1 - 2 \sin^2 \theta}{\sin \theta} + \frac{2 \sin \theta \cos \theta}{\cos \theta}$$

$$\frac{1 - 2 \sin^2 \theta}{\sin \theta} + \frac{2 \sin \theta}{\sin \theta}$$

$$\frac{1}{\sin \theta}$$

$$\csc \theta$$

$$\text{L.S.} = \text{R.S.}$$

Restrictions:

$$\sin \theta \neq 0, \cos \theta \neq 0$$



$$\theta \neq n\pi/2, n \in \mathbb{I}$$

$$\text{Pg 300} \\ \text{3a) } (c, s, e)$$

Chp 6 Review

1 Determine the general solution of $\sin 3x = \frac{1}{2}$

$\sin x = \frac{1}{2}$?



$x = 30^\circ, 150^\circ$ or $\frac{\pi}{6}, \frac{5\pi}{6}$ $\times \frac{1}{3}$
 $\therefore \sin 3x = \frac{1}{2}$ when $x = \frac{\pi}{18}, \frac{5\pi}{18}$

$x = \frac{\pi}{18} + n\frac{2\pi}{3}$ and $x = \frac{5\pi}{18} + n\frac{2\pi}{3}$ $n \in \mathbb{I}$
 OR
 $x = 10^\circ + n120^\circ$ and $x = 50^\circ + n120^\circ$ $n \in \mathbb{I}$

2 Solve $2\cos^2 x + \cos x - 1 = 0$ where $0 \leq x < 2\pi$

Let $y = \cos x$

$2y^2 + y - 1 = 0$

$2y^2 + 2y - y - 1 = 0$

$2y(y+1) - (y+1) = 0$

$(2y-1)(y+1) = 0$

$y = \frac{1}{2}, y = -1 \therefore \cos x = \frac{1}{2} \quad \cos x = -1$



$x = 60^\circ, 300^\circ$
 OR $x = \frac{\pi}{3}, \frac{5\pi}{3}$

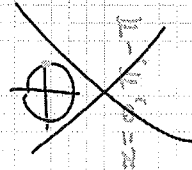
$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

3 Solve $\sin x \cot x - \sqrt{3} \sin x = 0$ $-2\pi < x < 2\pi$

$\sin x (\cot x - \sqrt{3}) = 0$

~~$\sin x = 0$~~ $\cot x = \frac{\cos x}{\sin x} = \sqrt{3}$

$\sin x \neq 0$



$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

3 Solve $\sin 2\theta - 2\sin^2 \theta = 0$ $-\pi \leq \theta \leq \pi$

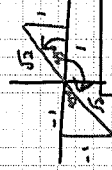
$2\sin\theta \cos\theta - 2\sin^2 \theta = 0$

$2\sin\theta (\cos\theta - \sin\theta) = 0$

$2\sin\theta = 0$ $\cos\theta - \sin\theta = 0$

$\sin\theta = 0$ $\cos\theta = \sin\theta$

\hookrightarrow When does $\cos\theta = \sin\theta$?



$\theta = 0, \pi, -\pi$

$\theta = 0, \pi, -\pi, \frac{\pi}{4}, \frac{3\pi}{4}$

5) Prove a) $\cos 2x = \cot x - \tan x$

$\cos^2 x - \sin^2 x$

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x} = 1 - \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x} = 1 - \tan^2 x$$

L.S. = R.S.

$\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos^2 x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$

b) $\frac{\cos 2\theta - \cos \theta}{\sin 2\theta + \sin \theta} = \frac{1 - \sec \theta}{\tan \theta}$

$$\frac{2\cos^2 \theta - \cos \theta - 1}{2\sin \theta \cos \theta + \sin \theta} = \frac{\cos \theta - 1}{\cos \theta}$$

$$\frac{(2\cos \theta + 1)(\cos \theta - 1)}{\sin \theta (2\cos \theta + 1)} = \frac{\cos \theta - 1}{\sin \theta}$$

L.S. = R.S.

$$\frac{\cos \theta - 1}{\cos \theta} = \frac{\cos \theta - 1}{\cos \theta}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta}$$

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*1-48
Not 15,16

c) $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$

$$\frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

L.S. = R.S.

4) Write as a single trig function and determine the exact value

a) $2\cos^2 \frac{\pi}{8} - 2\sin^2 \frac{\pi}{8}$

$2(\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8})$

$2 \cos 2(\frac{\pi}{8})$

$2 \cos \frac{\pi}{4}$

$2(\frac{1}{\sqrt{2}}) = \frac{2}{\sqrt{2}} = \sqrt{2}$

b) $4 - 8\cos^2 15^\circ$

$4(1 - 2\cos^2 15^\circ)$

$-4(2\cos^2 15^\circ - 1)$

$-4(\cos 2(15^\circ))$

$-4 \cos 30^\circ$

$-4(\frac{\sqrt{3}}{2}) = -2\sqrt{3}$

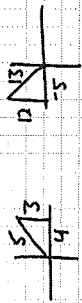
6) Given that $\sin P = \frac{3}{5}$ and $\cos Q = -\frac{5}{13}$ where $0 \leq P < \frac{\pi}{2}$ and $\frac{\pi}{2} \leq Q < \pi$, determine $\cos(P+Q)$.

$$\cos(P+Q) = \cos P \cos Q - \sin P \sin Q$$

$$= \cos P \left(-\frac{5}{13}\right) - \left(\frac{3}{5}\right) \sin Q$$

$$\sin P = \frac{3}{5}$$

$$\cos Q = -\frac{5}{13}$$



$$\cos P = \frac{4}{5} \quad \sin Q = \frac{12}{13}$$

$$\left(\frac{4}{5}\right)\left(-\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$

$$-\frac{20}{65} - \frac{36}{65}$$

$$\boxed{-\frac{56}{65}}$$

8) Which of the following expressions are equivalent to $\cos^2 \theta - \sin^2 \theta$?

A. $(\cos 2\theta)^2$

B. $\cos 4\theta$

C. $-\cos 2\theta$

D. $\cos 2\theta$

$$(\cos^2 \theta - \sin^2 \theta) (\cos^2 \theta + \sin^2 \theta)$$

$$(\cos 2\theta) (1)$$

D. $\boxed{\cos 2\theta}$

P. 305

* 1-48

Not 15, 16, 28