Chapter 6 Review

Section 6.1

1. Determine the restrictions

   a) $\frac{\sin x}{\cos x}$ b) $\frac{\sec x}{\sin x}$ c) $\frac{\tan x}{1-\cos x}$ d) $\frac{\cot x}{\sin x+1}$

2. Determine the restrictions for $\frac{1+\cos\theta}{\sin\theta}=\frac{\sin\theta}{1-\cos\theta}$

3. Simplify each expression

   a) $\frac{\cot x}{\csc x}$ b) $\cot x\sin x$ c) $\frac{1}{\cot x\sec x}$ d) $\frac{1-\tan x}{\cot x-1}$

4. Simplify each expression.

   a) $2(\csc^2 x-\cot^2 x)$ b) $\cot^2 x(\sec^2 x-1)$

   c) $\frac{\sin^2 x}{\cos^2 x}+\sin x\csc x$ d) $\frac{\cos x}{\sin x\cot x}$

   e) $\tan x\cos^2 x$ f) $\frac{1}{\sec^2 x}+\frac{1}{\csc^2 x}$

5. Are the following identities? a) $\sin^2 x\sec^2 x=\sec^2 x-1$

   b) $\frac{1}{\sec x}+\frac{1}{\csc x}=1$ c) $\cot x+\tan x=\csc x\cot x$

6. Simplify each expression a) $\frac{\sec x}{\sin x}-\frac{\sin x}{\cos x}$

   b) $\cos x+\tan x\sin x$ c) $\sin x+\cos x\cot x$

7. Verify $\sin^4 x-\cos^4 x=2\sin^2 x-1$ for $x=\frac{\pi}{6}$.

8. Verify $\sec x+\sec x\cos x=1+\sec x$. for $x=\frac{\pi}{4}$.

9. Given $\frac{\cos x}{\sec x-1}+\frac{\cos x}{\sec x+1}=2\cot^2 x$. a) Verify for $x=\frac{\pi}{6}$

   b) What are the restrictions of the equation in $0°\le x<360°$.

10. Algebraically change $\cos^2 x+\sin^2 x=1$ into $\cot^2 x+1=\csc^2 x$

Section 6.2 Extra Practice

1. Write each expression as a single trigonometric function.

   a) $\sin 28°\cos 35°+\cos 28°\sin 35°$

   b) $\cos 10°\cos 7°-\sin 10°\sin 7°$

   c) $\cos\frac{\pi}{12}\cos\frac{\pi}{4}+\sin\frac{\pi}{12}\sin\frac{\pi}{4}$

   d) $\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$

2. Simplify and then give an exact value for each expression.

   a) $\cos 25°\cos 5°-\sin 25°\sin 5°$

   b) $\sin 40°\cos 20°+\cos 40°\sin 20°$

   c) $\sin\frac{\pi}{3}\cos\frac{\pi}{6}+\cos\frac{\pi}{3}\sin\frac{\pi}{6}$

   d) $\cos\frac{7\pi}{12}\cos\frac{\pi}{3}+\sin\frac{7\pi}{12}\sin\frac{\pi}{3}$

3. Write each expression as a single trigonometric function.

   a) $2\sin\frac{\pi}{6}\cos\frac{\pi}{6}$ b) $\cos^2\frac{\pi}{3}-\sin^2\frac{\pi}{3}$

   c) $1-2\sin^2 15°$ d) $\frac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

4. Simplify each expression using a sum identity.

   a) $\sin(90°+A)$ b) $\cos(90°+A)$
   c) $\sin(\pi+A)$ d) $\cos(2\pi+A)$

5. Simplify each expression using a difference identity.

   a) $\sin(90°-A)$ b) $\sin(270°-A)$
   c) $\sin\left(\frac{\pi}{2}-A\right)$ d) $\cos\left(\frac{3\pi}{2}-A\right)$

6. Simplify each expression

   a) $\frac{\sin 2\theta}{2\sin\theta}$ b) $\cos 3x\cos x-\sin 3x\sin x$

   c) $\frac{\cos 2\theta-1}{2\sin\theta}$ d) $\frac{\sin^3 x}{\cos 2x-\cos^2 x}$

7. Determine the exact value of each expression.

   a) $\cos\frac{2\pi}{3}$ b) $\tan 15°$ c) $\sin 105°$ d) $\cos\frac{5\pi}{6}$

8. Determine whether each equation is true.

   a) $\cos 80°=\cos 75°\cos 5°-\sin 75°\sin 5°$
   b) $\cos(-24°)=\cos 16°-\cos 40°$ c) $\tan 70°=\frac{2\tan 30°}{1-\tan^2 70°}$

9. If ∠A &∠B are both in quadrant I, & $\sin A=\frac{3}{5}$ & $\cos B=\frac{5}{13}$ evaluate each of the following.

   a) $\cos(A-B)$ b) $\sin(A+B)$ c) $\cos 2A$ d) $\sin 2A$

10. If $\cos A=\frac{12}{13}$, and ∠A is in quadrant IV, find the exact value of sin 2A.

Section 6.3 Extra Practice

1. Simplify each expression

   a) $\frac{\sec x}{\tan x}$ b) $\frac{\cot^2 x}{1-\sin^2 x}$ c) $\frac{\csc x-\sin x}{\cot x}$

2. Factor and simplify each rational trigonometric expression.

3. Prove each identity for all permissible values of x.

a) $\csc^2 x(1 - \cos^2 x) = 1$ b) $(\tan x - 1)^2 = \sec^2 x - 2\tan x$

c) $\dfrac{\sin^2 x + \cos^2 x}{\sec x} = \cos x$

4. Prove each identity. a) $\dfrac{1 + \tan x}{1 + \cot x} = \tan x$

b) $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x} = \cot x$ c) $\dfrac{\cot x + \tan x}{\sec x} = \csc x$

5. Prove each identity. a) $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

b) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$ c) $\dfrac{\cos x + 1}{\sin x + \tan x} = \cot x$

6. Prove each identity. a) $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$

b) $\dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x}{1 - \cos x}$ c $\dfrac{\cos x}{\sec x - 1} + \dfrac{\cos x}{\sec x + 1} = 2\cot^2 x$

7. Prove the following a) $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$

b) $\dfrac{1 + \cos 2x}{\sin 2x} = \cot x$ c) $1 + \sin 2x = (\sin x + \cos x)^2$

d) $\sec^2 x = \dfrac{2}{1 + \cos 2x}$

8. Prove each identity a) $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

b) $\cos x + \cos x \tan^2 x = \sec x$

9. Consider the equation $\dfrac{\cos^2 x}{1 + 2\sin x - 3\sin^2 x} = \dfrac{1 + \sin x}{1 + 3\sin x}$.

Show that the equation is true for $x = 3.2$ radians.

10.a) Prove $\tan\theta = \dfrac{1 - \cos 2\theta}{\sin 2\theta}$. b) State all restrictions.

11. Prove the following identity. $1 + \sin 2x = (\sin x + \cos x)^2$

12. Prove the identity. $\cos 3x + 1 = 4\cos^3 x - 3\cos x + 1$

Section 6.4 Extra Practice

1. Solve each equation algebraically over the domain $0 \le x < 2\pi$.

a) $\sin 2x - \cos x = 0$ b) $\cos 2x = 0$

c) $2\cos^2 x - 1 = 0$ d) $\cos^2 x - 2 = \cos x$

2. Solve each equation over the domain $0° \le x < 360°$.

a) $\cos 2x = \cos 3x$ b) $2\cos^2 x - 5\sin x - 5 = 0$

c) $\cot^2 x = 0$

3. Rewrite each equation in terms of cosine only. Then, solve algebraically for $0 \le x < 2\pi$.

a) $\cos 2x - 5\cos x = 2$ b) $\cot^2 x + 2 = 0$ c) $1 + \cos x = 2\sin^2 x$

4. Solve $2\cos^2 x = 1$ over the domain $-180° \le x \le 180°$.

5. Solve $\tan^2 x + 2\tan x + 1 = 0$ over the domain $0 \le x < 2\pi$.

a) $\dfrac{\tan x - \tan x \sin^2 x}{\cos^2 x}$ b) $\dfrac{\sin^2 x + \sin x - 6}{5\sin x + 15}$

c) $\dfrac{\cos^2 x - 4}{7\cos x - 14}$ d) $\dfrac{\sin^2 x \tan x - \tan x}{\sin x \tan x + \tan x}$

6. Determine and correct the mistake in the following work.

$\sin 2x = 1$

$\sin x = \dfrac{1}{2}$

$x = 60°$ and $120°$

7. A student writes the general solution for $\sin 2x = 1$ over the domain $0 \le x < 2\pi$ as $\dfrac{\pi}{4} + 2\pi n$, $\dfrac{5\pi}{4} + 2\pi n$; $n \in I$.

a) What error did the student make?

b) Write the correct general solution for this equation.

8. Solve $\cos x - 2\sin x\cos x = 0$ over the domain $0 \le x < 2\pi$.

9. Solve $(\sin x - 1)(\tan x - 1) = 0$ for all values of x.

10. Solve $2\cos 2x + 1 = 0$Give the general solution in degrees.

Chapter 6 Answers Section 6.1

1. a) $x \ne \dfrac{\pi}{2} + \pi n$; $n \in I$ b) $x \ne \pi n$; $n \in I$ and $x \ne \dfrac{\pi}{2} + \pi n$; $n \in I$

c) $x \ne 2\pi n$; $n \in I$ and $x \ne \dfrac{\pi}{2} + \pi n$; $n \in I$ d) $x \ne \dfrac{\pi}{2}n$; $n \in I$

2. $\theta \ne \pi n$; $n \in I$ 3. a) $\cos x$ b) $\cos x$ c) $\sin x$ d) $\tan x$

4. a) 2 b) 1 c) $\sec^2 x$ d) 1 e) $\sin x\cos x$ f) 1 5. a) Yes b) No c) No

6. a) $\cot x$ b) $\sec x$ c) $\csc x$

7. Left side $= \sin^4\left(\dfrac{\pi}{6}\right) - \cos^4\left(\dfrac{\pi}{6}\right)$ Right side $= 2\sin^2\left(\dfrac{\pi}{6}\right) - 1$

$= \dfrac{1}{16} - \dfrac{9}{16}$ $\quad$ $= -\dfrac{1}{2}$

$= -\dfrac{1}{2}$ $\quad$ = Left side

8. Left side $= \sec\left(\dfrac{\pi}{4}\right) + \sec\left(\dfrac{\pi}{4}\right)\cos\left(\dfrac{\pi}{4}\right)$ Right side $= 1 + \sec\left(\dfrac{\pi}{4}\right)$

$= \dfrac{1}{\cos\left(\dfrac{\pi}{4}\right)} + \dfrac{\cos\left(\dfrac{\pi}{4}\right)}{\cos\left(\dfrac{\pi}{4}\right)}$ $\quad$ $= 1 + \dfrac{1}{\cos\left(\dfrac{\pi}{4}\right)}$

$= 1 + \dfrac{2}{\sqrt{2}}$

$= \dfrac{2}{\sqrt{2}} + 1$ $\quad$ = Left side

9. a) $LS = \cos\left(\dfrac{\pi}{6}\right) \div \left(\sec\left(\dfrac{\pi}{6}\right) - 1\right) + \cos\left(\dfrac{\pi}{6}\right) \div \left(\sec\left(\dfrac{\pi}{6}\right) + 1\right)$

$= \dfrac{\sqrt{3}}{2} \div \left(\dfrac{2}{\sqrt{3}} - 1\right) + \dfrac{\sqrt{3}}{2} \div \left(\dfrac{2}{\sqrt{3}} + 1\right)$

9b) $x \neq 0°, 180°$ 10.

$$\cos^2 x + \sin^2 x = 1$$
$$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$
$$\cot^2 x + 1 = \csc^2 x$$

Section 6.2

1. a) $\sin 63°$ **b)** $\cos 17°$ **c)** $\cos\left(-\frac{\pi}{6}\right)$ **d)** $\sin\left(\frac{\pi}{12}\right)$

2. a) $\frac{\sqrt{3}}{2}$ **b)** $\frac{\sqrt{3}}{2}$ **c)** 1 **d)** $\frac{1}{\sqrt{2}}$ **3. a)** $\sin\frac{\pi}{3}$ **b)** $\cos\frac{2\pi}{3}$ **c)** $\cos 30°$

3 **d)** $\tan\frac{\pi}{3}$ **4. a)** cos A **b)** −sin A **c)** −sin A **d)** cos A

5. a) cos A **b)** −cos A **c)** cos A **d)** −sin A

6. a) $\cos\theta$ **b)** $\cos(4x)$ **c)** $-\sin\theta$ **d)** $-\sin\theta$

7. a) $-\frac{1}{2}$ **b)** $2-\sqrt{3}$ **c)** $\frac{\sqrt{3}+1}{2\sqrt{2}}$ **d)** $-\frac{\sqrt{3}}{2}$

8. a) true **b)** false **c)** true **d)** false

9. a) $\frac{56}{65}$ **b)** $\frac{63}{65}$ **c)** $\frac{7}{25}$ **d)** $\frac{24}{25}$ **10.** $-\frac{120}{169}$

6.3 Extra Practice

1. a) $\frac{1}{\sin x}$ **b)** $\frac{1}{\sin^2 x}$ **c)** $\cos x$ **2. a)** $\tan x$ **b)** $\frac{\sin x - 2}{5}$ **c)** $\frac{\cos x + 2}{7}$ **d)** $\sin x - 1$ **3a)**

$$\text{Left side} = \csc^2 x\left(1-\cos^2 x\right)$$
$$= \frac{1}{\sin^2 x}(\sin^2 x)$$
$$= 1$$
$$= \text{Right side}$$

3b)
$$\text{Left side} = (\tan x - 1)^2$$
$$= \tan^2 x - 2\tan x$$
$$= \frac{\sin^2 x - 2\sin x c}{\cos^2 x}$$

3c) Example:

$$\text{Right side} = \cos x$$
$$\text{Left side} = \frac{\sin^2 x + \cos^2 x}{\sec x}$$
$$= 1 \div \frac{1}{\cos x}$$
$$= \cos x$$
$$= \text{Right side}$$

4. a) Example: right side = $\tan x$

$$\text{Left side} = \frac{1+\tan x}{1+\cot x}$$
$$= 1 + \frac{\sin x}{\cos x} \div \left(1 + \frac{\cos x}{\sin x}\right)$$
$$= \frac{\cos x + \sin x}{\cos x} \times \frac{\sin x}{\sin x + \cos x}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$
$$= \text{Right side}$$

b) Example:

Right side = $\cot x$

$$\text{Left side} = \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}$$
$$= \frac{1}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x}$$
$$= \frac{\cos^2 x}{\sin x \cos x}$$
$$= \cot x$$
$$= \text{Right side}$$

$$RS = 2\cot^2\left(\frac{\pi}{6}\right)$$
$$= \frac{2}{\tan^2\left(\frac{\pi}{6}\right)}$$
$$= 2 \div \frac{1}{3}$$
$$= 6$$
$$= LS$$

c) Example:

$$\text{Right side} = \csc x$$
$$= \frac{1}{\sin x}$$
$$\text{Left side} = \frac{\cot x + \tan x}{\sec x}$$
$$= \cos x\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)$$
$$= \frac{\cos^2 x + \sin^2 x}{\sin x}$$
$$= \frac{1}{\sin x}$$
$$= \text{Right side}$$

5a) Example:

$$\text{Left side} = \frac{\csc x + \cot x}{\tan x + \sin x}$$
$$= \frac{1+\cos x}{\sin x} \div \frac{\sin x + \sin x \cos x}{\cos x}$$
$$= \frac{1+\cos x}{\sin x} \times \frac{\cos x}{\sin x(1+\cos x)}$$
$$= \frac{\cos x}{\sin^2 x}$$

$$\text{Right side} = \cot x \cdot$$
$$= \frac{\cos x}{\sin^2 x}$$
$$= \text{Left s}$$

5b)
$$\text{Right side} = \tan x$$
$$\text{Left side} = \frac{\sin x + \tan x}{\cos x + 1}$$
$$= \frac{\sin x \cos x + \sin x}{\cos x} \times \frac{1}{\cos x}$$
$$= \frac{\sin x(\cos x + 1)}{\cos x} \times \frac{1}{\cos x +}$$
$$= \tan x$$
$$= \text{Right side}$$

c)
$$\text{Right side} = \cot x$$
$$\text{Left side} = \frac{\cos x + 1}{\sin x + \tan x}$$
$$= (\cos x + 1) \times \frac{\cos x}{\sin x(\cos x + 1)}$$
$$= \cot x$$
$$= \text{Right side}$$

6. a)
$$\text{Right side} = \frac{1+\sin x}{\cos x}$$
$$\text{Left side} = \frac{\cos x}{1-\sin x}$$
$$= \frac{\cos x(1+\sin x)}{(1-\sin x)(1+\sin x)}$$
$$= \frac{\cos x(1+\sin x)}{1-\sin^2 x}$$
$$= \frac{\cos x(1+\sin x)}{\cos^2 x}$$
$$= \frac{1+\sin x}{\cos x}$$
$$= \text{Right side}$$

b)
$$\text{Left side} = \frac{1+\cos x}{\sin x}$$
$$\text{Right side} = \frac{\sin x}{1-\cos x} \times \frac{1+\cos x}{1+\cos x}$$
$$= \frac{\sin x(1+\cos x)}{\sin^2 x}$$
$$= \frac{1+\cos x}{\sin x}$$

c) Left side $= \dfrac{\cos x}{\sec x - 1} + \dfrac{\cos x}{\sec x + 1}$

$= \cos x \div \dfrac{1 - \cos x}{\cos x} + \cos x \div \dfrac{1 + \cos x}{\cos x}$

$= \dfrac{\cos^2 x}{1 - \cos x} + \dfrac{\cos^2 x}{1 + \cos x}$

$= \dfrac{2\cos^2 x}{1 - \cos^2 x}$

$= \dfrac{2\cos^2 x}{\sin^2 x}$

= Right side

Right side $= 2\cot^2 x$

$= \dfrac{2\cos^2 x}{\sin^2 x}$

7. a) Example: Right side $= \cos^2 x - \sin^2 y$

Left side $= \cos(x + y)\cos(x - y)$

$= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x(1 - \sin^2 y) - \sin^2 y(1 - \cos^2 x)$

$= \cos^2 x - \sin^2 y \cos^2 x - \sin^2 y + \sin^2 y \cos^2 x$

$= \cos^2 x - \sin^2 y$

= Right side

b) Example:

Right side $= \cot x$

$= \dfrac{\cos x}{\sin x}$

Left side $= \dfrac{1 + \cos 2x}{\sin 2x}$

$= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$

$= \dfrac{\cos x}{\sin x}$

= Right side

c) Example:

Left side $= 1 + \sin 2x$

$= 1 + 2\sin x \cos x$

= Right side

$= \sin^2 x + 2\sin x \cos x + \cos^2 x$

$= 1 + 2\sin x \cos x$

d)

Right side $= \dfrac{2}{1 + \cos 2x}$

$= \dfrac{2}{1 + 2\cos^2 x - 1}$

$= \dfrac{1}{\cos^2 x}$

Left side $= \sec^2 x$

$= \dfrac{1}{\cos^2 x}$

= Right side

8. a)

Example:

Left side $= \sec^4 x - \sec^2 x$

$= \dfrac{1}{\cos^2 x}\left(\dfrac{1 - \cos^2 x}{\cos^2 x}\right)$

$= \dfrac{\sin^2 x}{\cos^4 x}$

Right side $= \tan^4 x + \tan^2 x$

$= \tan^2 x(\tan^2 x + 1)$

$= \dfrac{\sin^2 x}{\cos^2 x}\left(\dfrac{1}{\cos^2 x}\right)$

$= \dfrac{\sin^2 x}{\cos^4 x}$

= Left side

b)

Right side $= \sec x$

$= \dfrac{1}{\cos x}$

= Left side

Left side $= \cos x + \cos x \tan^2 x$

$= \dfrac{\cos^3 x + \cos x \sin^2 x}{\cos^2 x}$

$= \dfrac{\cos x(\cos^2 x + \sin^2 x)}{\cos^2 x}$

$\dfrac{1}{\cos x}$

9Verify for $x = 3.2$:

Left side $= \dfrac{\cos^2 3.2}{1 + 2\sin 3.2 - 3\sin^2 3.2}$

≈ 1.14153

Right side $= \dfrac{1 + \sin}{1 + 3\sin}$

≈ 1.14153

10. a)

Left side $= \tan\theta$

Right side $= \dfrac{1 - \cos 2\theta}{\sin 2\theta}$

$= \dfrac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

= Left side

b) $\theta \neq \dfrac{n\pi}{2}; n \in I$

11. Example:

Left side $= 1 + \sin 2x$

$= 1 + 2\sin x \cos x$

Right side $= (\sin x + \cos x)^2$

$= \sin^2 x + 2\sin x \cos x + \cos^2$

$= 1 + 2\sin x \cos x$

= Left side

12. Example:

Left side $= \cos 3x + 1$

$= \cos(2x + x) + 1$

$= \cos 2x \cos x - \sin 2x \sin x + 1$

$= \cos x(2\cos^2 x - 1) - 2\sin x(\sin x \cos x) + 1$

$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x + 1$

$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x) + 1$

$= 4\cos^3 x - 3\cos x + 1$

Right side $= 4\cos^3 x - 3\cos x + 1$

= Left side

Section 6.4

1. a) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **b)** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$ **c)** $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$ **d)** π **2. a)** 0°, 72°, 144°, 216°, 288° **b)** 270° **c)** 90°, 270°

3. a) $\frac{2\pi}{3}, \frac{4\pi}{3}$ **b)** no solution **c)** $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

4. −135°, −45°, 45°, 135° **5.** $\frac{3\pi}{4}, \frac{7\pi}{4}$

6. The error was in dividing 1 by 2.

$$\begin{aligned} \sin 2x &= 1 \\ 2x &= 90° \\ x &= 45° \end{aligned}$$

7. a) The student used 2π rather than π; because the equation is $\sin 2x = 1$, the period of the function is π.

b) $\frac{\pi}{4} + \pi n, \frac{5\pi}{4} + \pi n;\ n \in I$

8. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

9. $\frac{\pi}{2} + 2\pi n, \frac{\pi}{4} + \pi n;\ n \in I$

10. 60°, 120°, 240°, 300° Gen Sol: $60° + 180°n$, $120° + 180°n$; $n \in I$