Transformations Practice Exam - ANSWERS

Use this sheet to record your answers

1. A

NR 1. 5

2. C

3. B

NR 2. 4

4. B

5. A

6. B

7. D

8. C

9. A

10. C

11. C

12. A

NR 3. 231

13. D

14. B

15. A

16. C

17. A

NR 4. 3

NR 5. 8212

18. D

19. D

20. B

NR 6. 57

21. C

22. D

23. D

24. B

25. B

26. A

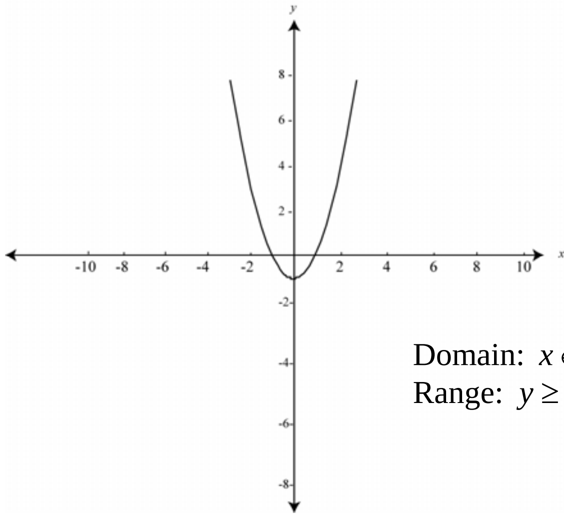
27. D

28. B

29. **C**

Transformations Practice Exam - Answers

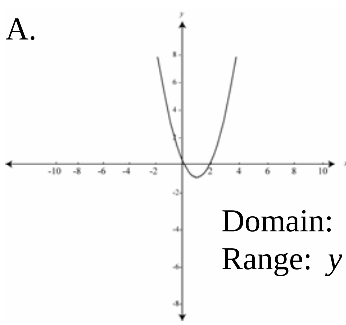
1. If $f(x) = x^2 - 1$, then it is a parabola as shown below.



Domain: $x \in R$
Range: $y \geq -1$

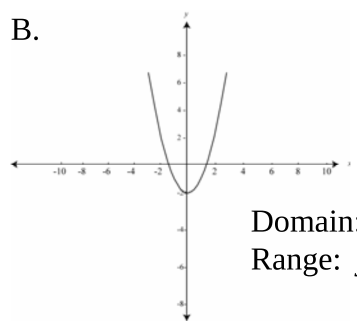
Look at the graph in each response:

A.



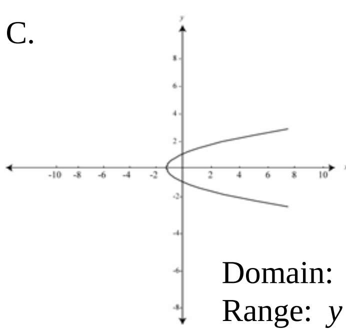
Domain: $x \in R$
Range: $y \geq -1$

B.



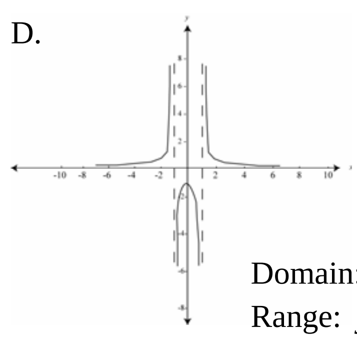
Domain: $x \in R$
Range: $y \geq -2$

C.



Domain: $x \geq -1$
Range: $y \in R$

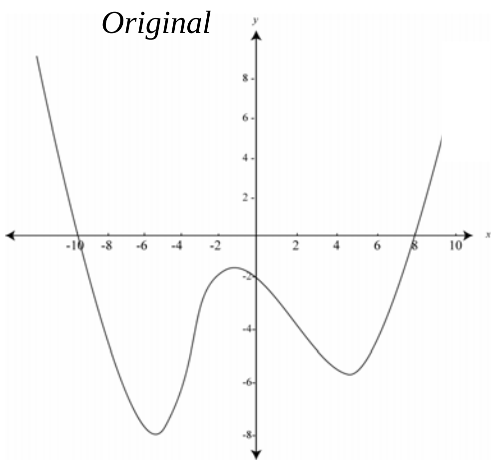
D.



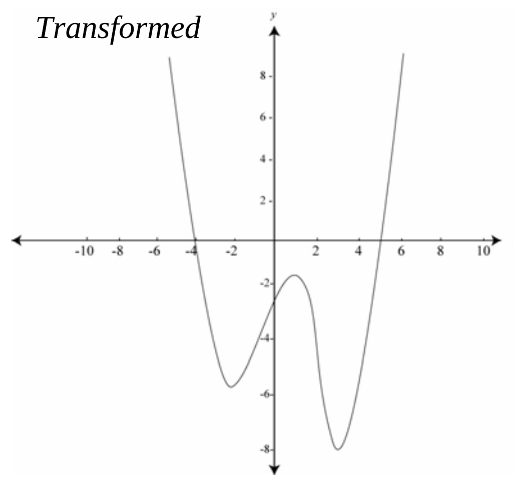
Domain: $x \in R,\ x \neq \pm 1$
Range: $y \in R,\ y \neq 0$

The only graph with the same domain and range as the original is **A**.

NR1) Draw the graph of the transformation $y = f(-2x)$. Multiply the x-values by $\frac{1}{2}$ and reflect across the y-axis.

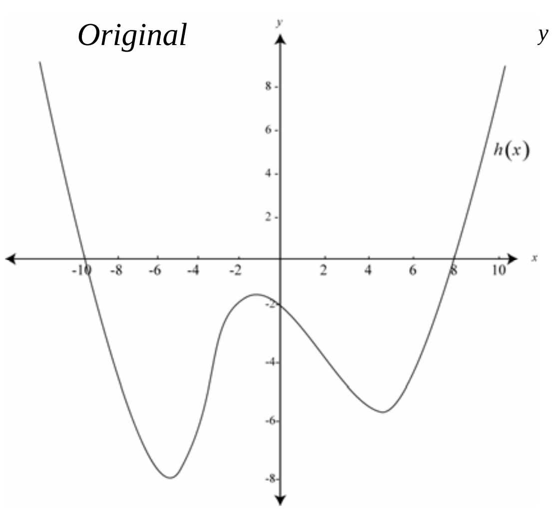


$y = f(-2x)$ →

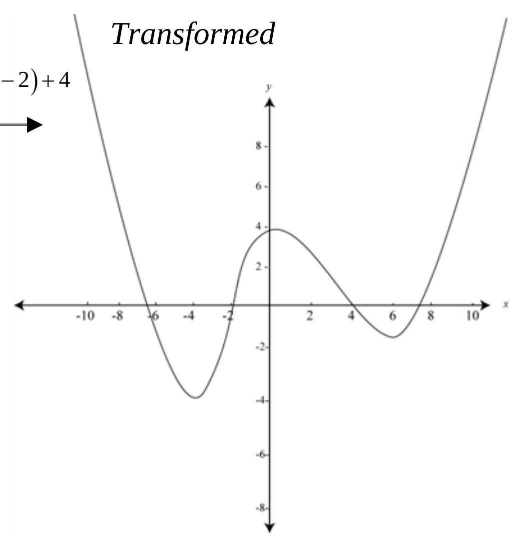


As can be seen in the transformed graph, the largest x-intercept is **5**.

2. You can rewrite this as: $y = f(x-2)+4$.
Move the original right 2 units, and up 4 units.

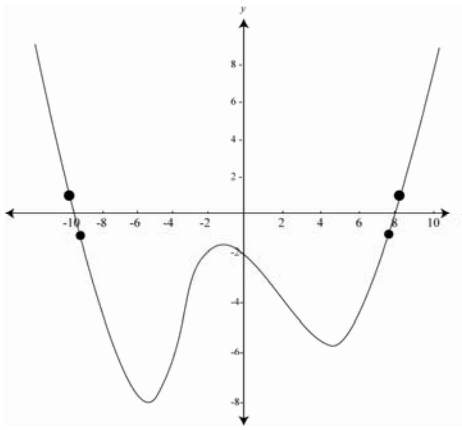


$y = f(x-2)+4$ →



The range of the transformed graph is $y \geq -4$
The answer is **C**.

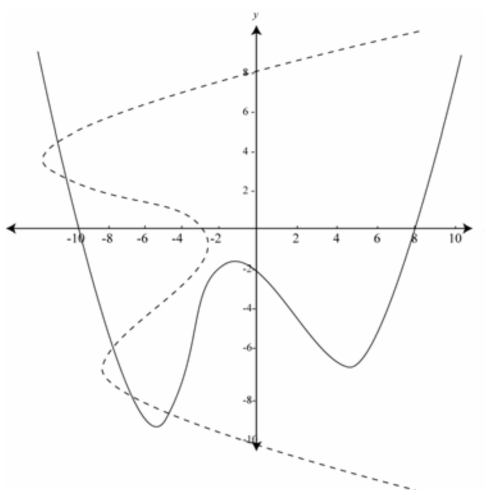
3. The invariant points on the graph of a reciprocal occur when the original has a y-value equal to 1 or -1.



This happens on the graph four times, at the points indicated.
(The relative maximum in the middle is too low and does not reach a y-value of -1.)
The answer is **B.**

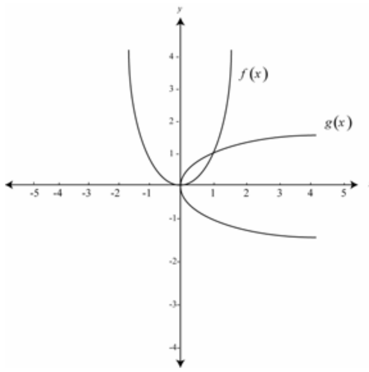
NR2. The y-intercept is three units away from the line $y = -5$. When those three units are stretched by a factor of 3, the new point is nine units away from the line, at the point (0, 4).
The answer is **4.**

4. The inverse graph is reflected about the line $y = x$.



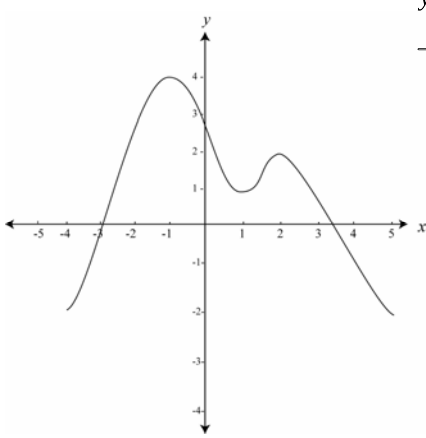
The inverse graph does not pass the vertical line test, therefore it is not a function.
The answer is **B**.

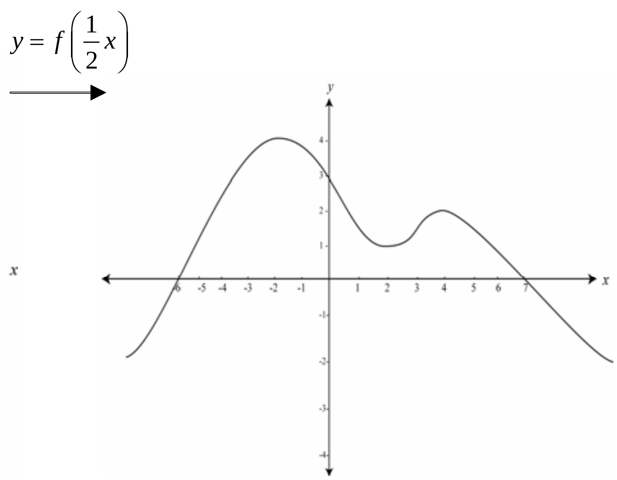
5. The graph of the inverse is reflected in the line $y = x$. In the case of a parabola with the vertex at the origin, there must be overlap to properly draw in the graph.



The answer is **A**.

6. The graph of $y = f\left(\frac{1}{2}x\right)$ is drawn by applying a horizontal stretch by a factor of 2 to the original graph.

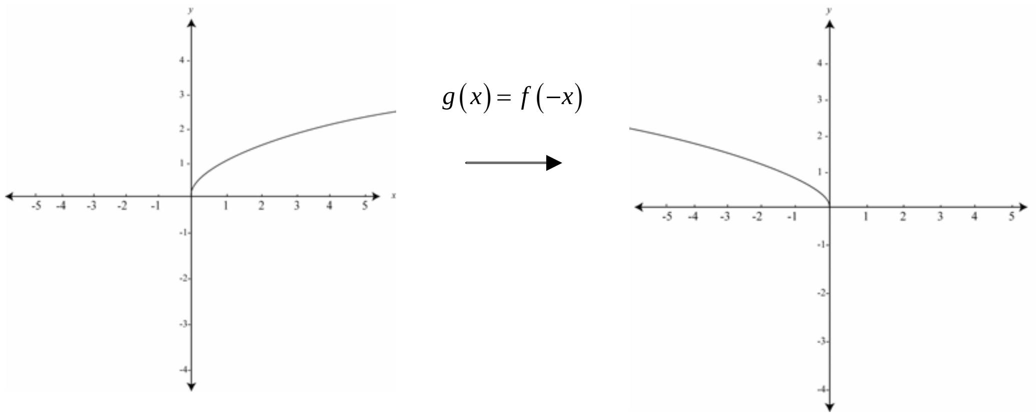




The answer is **B**.

7. The graph of $y = -2f(x+5)$ has the following transformations: a vertical stretch by a factor of 2, a reflection in the x-axis, and a shift of five units left.
The answer is **D**.

8. The graph of $g(x) = f(-x)$ is a reflection in the *y*-axis.

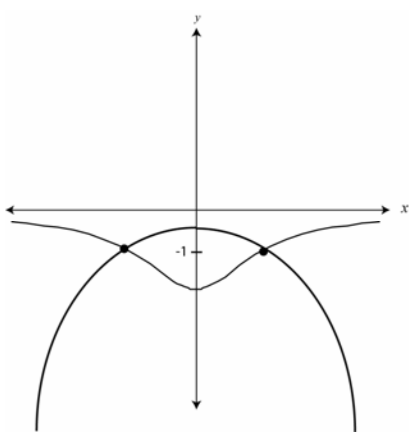


The graph of the transformation has a range of $y \geq 0$
The answer is **C**.

9. Apply the transformation $y = f(2x+4)$ to the point (8, -5)
Rewrite as $y = f\left[2(x+2)\right]$

Horizontal stretch by a factor of $\frac{1}{2}$ gives (4, -5) → (*multiply x – values by ½*)
Shift two units left: (2, -5) → (*subtract 2 from the x – coordinate*)
The answer is **A**.

10. In a reciprocal graph, invariant points occur when the *y*-value is ±1.



The graph has two invariant points.
The answer is **C**

11. The transformation $f(x+1)-2$ tells you to replace x with x + 1.

Original: $f(x)=x^2-2$

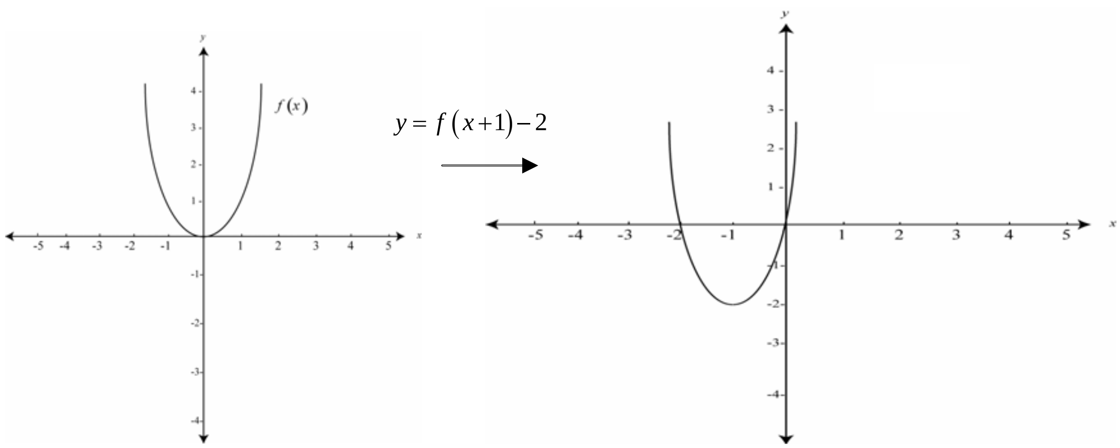
Transformed: $f(x+1)=(x+1)^2-2$

Replace x with x+1

The result is $(x+1)^2-2$

The answer is **C**.

12. Rewrite as $y=f(x+1)-2$ and apply the transformation



From the graph, it can be seen that the domain, but not the range, is the same.

The answer is **A**.

NR3. Quadrant II is a reflection in the y – axis, and that is equation 2.
Quadrant III is a reflection in both axis, and that is equation 3.
Quadrant IV is a reflection in the x – axis, and that is equation 1.
The answer is **231**.

13. The graph of $f(x)$ is stretched by a factor of $\frac{1}{2}$ about the line $y = 2$, then is reflected across the y-axis.
The answer is **D**.

14. The graph of $f(x)$ is moved one unit right, then 8 units down.
The answer is **B**.

15. Piece the transformation together starting with $y = f(x)$

A horizontal stretch of 3 means we use $\frac{1}{3}$ with x: $y = f\left(\frac{1}{3}x\right)$

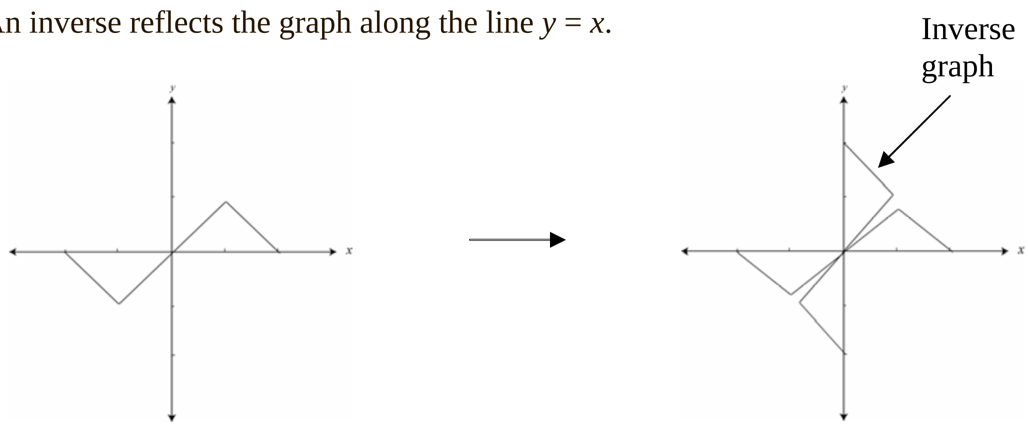
A reflection in the x-axis means we put a negative outside: $y = -f\left(\frac{1}{3}x\right)$

Translated 4 right and 2 up gives $y = -f\left(\frac{1}{3}(x-4)\right) + 2$

The answer is **A**.

16. If the graph is reflected in the y-axis, then points on the y-axis don't change. The only point on the y-axis is III.
The answer is **C**.

17. An inverse reflects the graph along the line $y = x$.



The graph that gives the new shape is **A**.

NR4. Write the function as $y = b\left(2x^3 - 4x^2 + 3x - 5\right)$, then plug in the point given.

$$-129 = b\left[2(-2)^3 - 4(-2)^2 + 3(-2) - 5\right]$$
$$-129 = b\left[2(-8) - 4(4) + 3(-2) - 5\right]$$
$$-129 = b\left[-8 - 16 - 6 - 5\right]$$
$$-129 = -43b$$
$$b = 3$$

The answer is **3**.

NR5. Rewrite as: $y = 2f(x) + 4$. Apply the transformation by multiplying the ***y*-values** by two, then adding four.
(-2, 2) → (-2, 8)
(1, -1) → (1, 2)
(5, 4) → (5, 12)

The answer is **8212**

18. The point (2, 2) lies on the line $y = x$, so it is invariant in an inverse graph.
The answer is **D**.

19. Vertical asymptotes occur at the *x*-intercepts of the original graph.
Therefore, $x = 8$ will be an asymptote.
The answer is **D**.

20. Do exactly as the question says: Replace *y* with $\frac{1}{3}y$

$$y = f(x)$$
$$\frac{1}{3}y = f(x)$$

Then cross multiply to get $y = 3f(x)$
This is a vertical stretch by a factor of 3.
The answer is **B**.

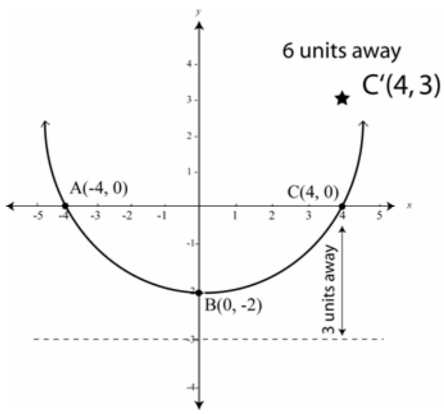
NR6. Think of the point (7, 8) as being on the graph of $g(x) = f(x-7)+8$

If the graph is further transformed by moving it two units left and one unit down, then the point (7, 8) becomes (5, 7).

The new equation of the transformation is $g(x) = f(x-5)+7$

The answer is **57**

21. Point C is three units above y = -3, so when it is stretched vertically by a factor of 2 about that line, the new coordinate is (4, 3)



The answer is **C**.

22. The vertical asymptotes in the graph of $\dfrac{1}{f(x)}$ are found at the x-intercepts,

There are four x-intercepts, and therefore there are four vertical asymptotes.
The answer is **D**.

23. The invariant points in the graph of $\dfrac{1}{f(x)}$ occur when $y = \pm 1$

There are four points where $y = \pm 1$, so there are four invariant points.
The answer is **D**.

24. Rewrite $g(x) = f(2x-4)$ as $g(x) = f\left[2(x-2)\right]$

There is a horizontal stretch by a factor of $\dfrac{1}{2}$, and a shift of 2 units right.

The point (-2, 1) becomes (1, 1)
The answer is **B**.

25. If the domain is $x \le 3$, and the graph is shifted 10 units left, the new domain is $x \le -7$
The answer is **B**.

26. Rewrite $y = f(3x-6)-1$ as $y = f\left[3(x-2)\right]-1$

The point (-3, 4) is transformed as follows:

Horizontal stretch by $\frac{1}{3}$ → (-1, 4)
Two units right and one unit down → (1, 3)
The answer is **A**.

27. Write the function as $y = bf(x)$, then plug in the information

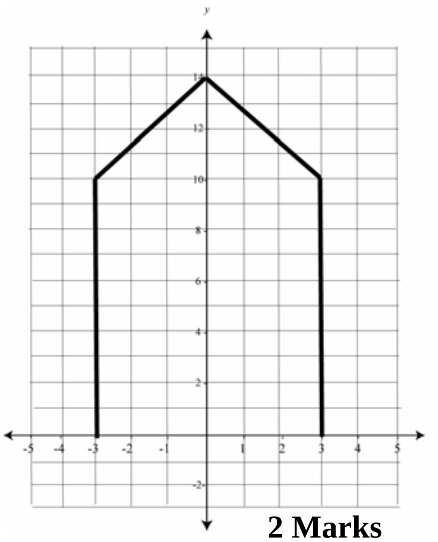
$$y = b\left(x^2 - 5x + 6\right)$$
$$15 = b\left(8^2 - 5(8) + 6\right)$$
$$15 = 30b$$
$$b = \frac{1}{2}$$

The answer is **D**.

28. The inverse graph of $y = (x+1)^2$ is a horizontal parabola, and does not pass the vertical line test. The inverse is not a function.
The answer is **B**.

29. The vertex of (0, 0) is 2 units away from the line x = 2, so when the stretch of $\frac{1}{2}$ is applied, it will only be one unit away from that line. The new vertex is at (1, 0)
The answer is **C**.

Written Response 1:



2 Marks

Use $p = 0$, and $q = 14$ for the coordinates of the vertex.

To determine the value of b in the equation $y = b|x - p| + q$, you can pick another point on the triangular arch, such as (3, 10), and use this for x & y.

Plugging in all the values into the equation gives

$$10 = b|3-0| + 14$$
$$10 = 3b + 14$$
$$-4 = 3b$$
$$b = -\frac{4}{3}$$

The equation is $y = -\frac{4}{3}|x| + 14$

2 Marks

Raising the height of the arch is the same as a vertical stretch, so it should result in the b value getting larger.

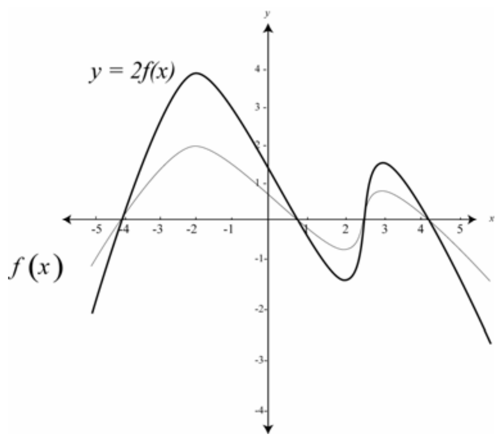
There is no shift left or right, so the p value stays the same.

The vertex gets higher, so the q value should get bigger.

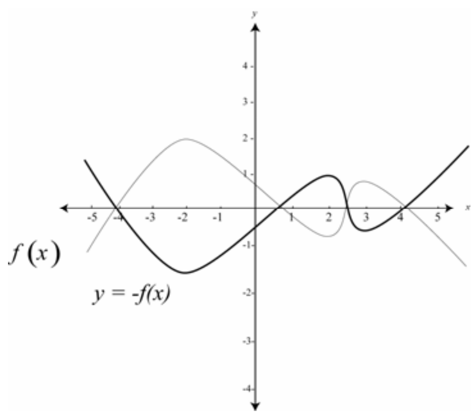
2 Marks

Written Response 2:

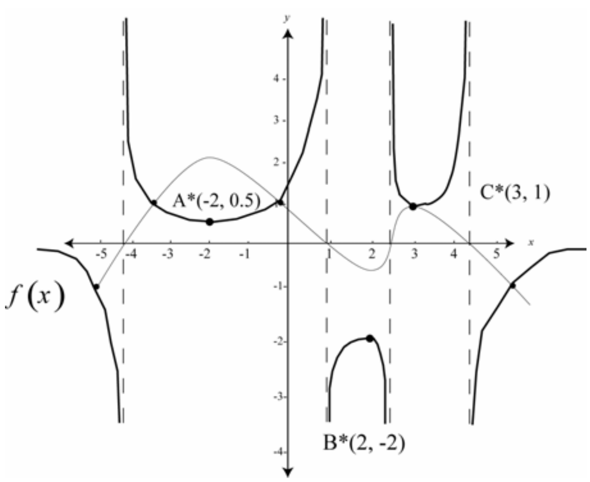
Vertical stretch by a factor of 2
(about the x-axis)



Reflection in the x - axis



Reciprocal transformation



2 Marks Each.

Written Response 3:

$y = -f(x)$ → B, E

$y = f(-x)$ → D

$y = f^{-1}(x)$ → D

$y = \frac{1}{f(x)}$ → A, C, F. G

V.S. about $y = \frac{1}{2}$ → D

$y = -f(-x)$ → None

1 Mark Each