Pure Math 30

EXPLAINED!

Diploma Style
Practice Exam

Trigonometry I

-- Answers --

Formulas

These are the formulas for Trig I you will be given on your diploma.

$$a = r\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

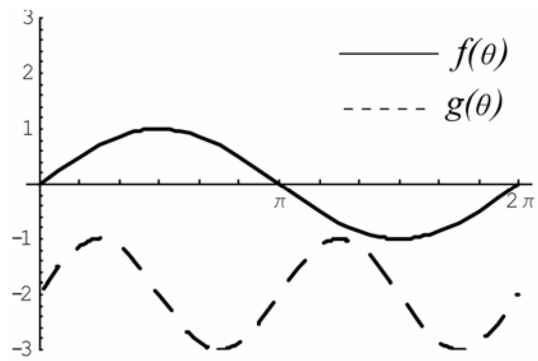
Answer Sheet

1. A	**NR 2)** 0.33	**19.** B	**29.** D
2. C	**NR 3)** 0.28	**20.** D	**30.** C
NR 1) 2.83	**11.** C	**21.** C	**31.** D
3. C	**12.** B	**22.** D	**32.** A
4. B	**13.** C	**23.** D	
5. A	**14.** A	**24.** D	
6. A	**NR 4)** 111	**NR 5)** 14.7	
7. C	**15.** D	**25.** B	
8. C	**16.** C	**26.** B	
9. B	**17.** B	**27.** C	
10. D	**18.** B	**28.** D	

1. Draw the following graphs in your graphing calculator:

$$f(\theta) = y_1 = \sin\theta$$
$$g(\theta) = y_2 = \sin 2\theta - 2$$

The transformations applied to the original graph are a horizontal stretch by a factor of $\frac{1}{2}$, and a shift down by two units.



As can be seen on the graph, the new range for $g(\theta)$ is $-3 \le y \le -1$.

The answer is **A**.

2. First state all the information you know:

- The length of arc is 8952 km.
- The angle is 55˚. However, we never use degrees in the arc length formula, so this must be converted to radians. $55^0 \bullet \frac{\pi}{180^0} = 0.9599 \text{ rad}$

Now use the arc length formula $a = r\theta$ to solve for the radius.

$$a = r\theta$$
$$r = \frac{a}{\theta} = \frac{8952}{0.9599} = 9325.97 \text{ km}$$

To find the height of the satellite above the surface of the planet, take the value you just found (*height of satellite from the centre of the planet*) and subtract the radius of the planet.

9325.97 km – 5000 km = 4325.97 km

The answer is **C**.

NR #1) When a question gives a point, such as $\left(\frac{\pi}{2},-2\right)$, these values can be put in for *x* and *y*.

In the context of this question, x and y are called θ *and* $f\left(\theta\right)$.

$$f(\theta)=a\cos\left(\theta-\frac{\pi}{4}\right)-4$$

$$-2=a\cos\left(\frac{\pi}{2}-\frac{\pi}{4}\right)-4 \qquad \text{Insert the given point.}$$

$$-2=a\cos\left(\frac{\pi}{4}\right)-4 \qquad \frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4},\ \text{or thinking in degrees, } 90^0-45^0=45^0$$

$$2=a\cos\left(\frac{\pi}{4}\right) \qquad \text{Bring the -4 over to the other side}$$

$$2=a\frac{\sqrt{2}}{2} \qquad \text{Since } \cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$4=a\sqrt{2}$$

$$a=2.83$$

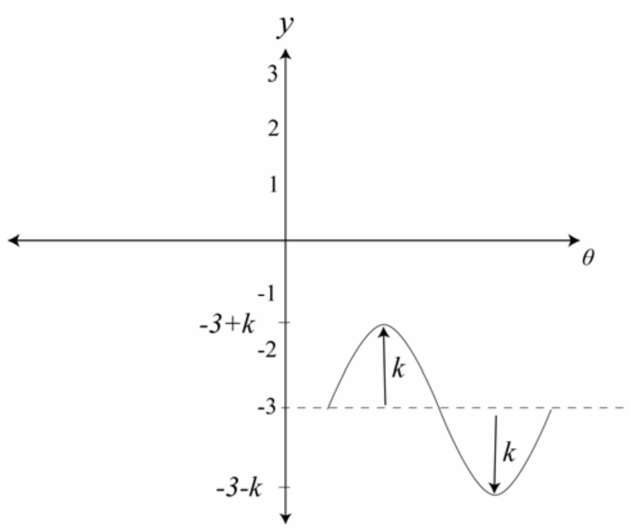
The value of *a*, to the nearest hundredth, is **2.83**

3. Look at the graph on the right. Notice that the midline has been shifted three units down, and the amplitude is *k*.
That means:

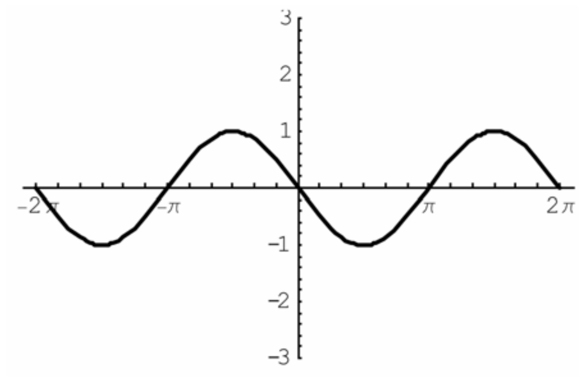
Minimum = -3 - *k*
Maximum = -3 + *k*

The range of the graph is
$-3-k\le f\left(\theta\right)\le -3+k$

The answer is **C**.



4. Draw the graph of $y = \cos\left(\theta + \frac{\pi}{2}\right)$



This graph is identical to the graph of $\sin\theta$, reflected in the *x*-axis.

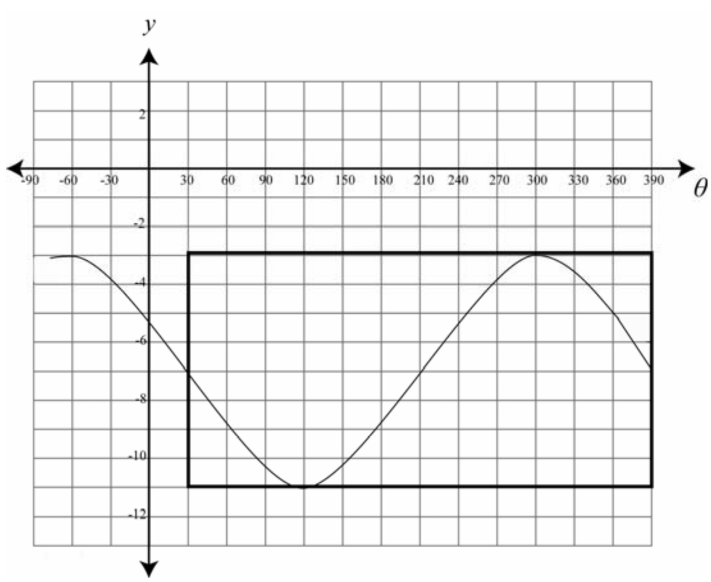
The answer is **B**.

5. At the *y*-intercept, $\theta = 0$.

$$
\begin{aligned}
f(\theta) &= -3\cos\left(k\theta + \frac{\pi}{2}\right) - b \\
&= -3\cos\left(k(0) + \frac{\pi}{2}\right) - b \\
&= -3\cos\left(\frac{\pi}{2}\right) - b \\
&= -3(0) - b \quad \left[\text{since } \cos\left(\frac{\pi}{2}\right) = 0\right] \\
&= -b
\end{aligned}
$$

The answer is **A.**

6. By inspection, the midline is at -7, and the amplitude is 4.



To determine the correct answer, it is necessary to check each of the possible answers to see which one actually matches the graph provided.

The only equation which is a correct match is
$f(\theta) = -4\sin\left(\theta - 30^0\right) - 7$

The answer is **A**.

7. *a* and *d* will be the same.

The *b*-value will be the same. (see below for proof)

Degree Mode: $b = \frac{360^0}{P} = \frac{360^0}{360^0} = 1$

Radian Mode: $b = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$

Thus, the only value which will change is the *c*-value, which changes from 30˚ to $\frac{\pi}{6} = 0.5236$

The answer is **C.**

8. At point A, the *y*-value of both graphs is equal to *m*.

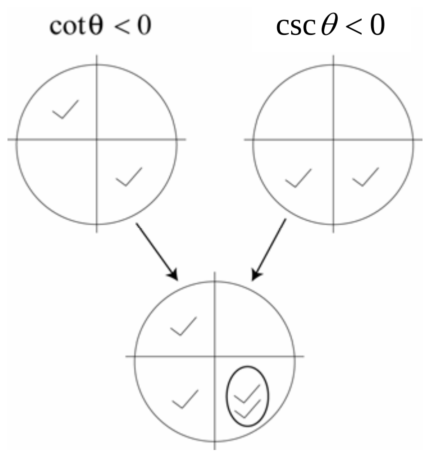
Therefore, $f(k) + g(k) = m + m = 2m$

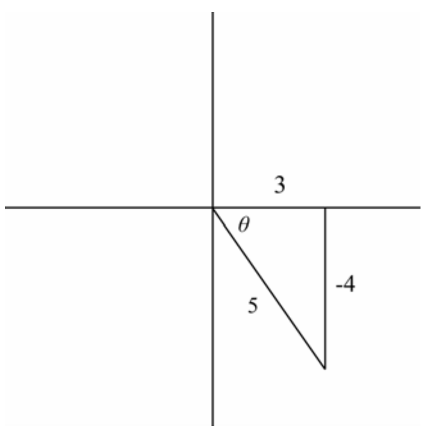
The answer is **C**.

9. First determine the quadrant the angle is found in.

Based on the diagrams to the right, the angle is in Quadrant IV.

Now draw in the triangle and use Pythagoras to solve for the hypotenuse.





From the triangle, $\sin\theta = \frac{-4}{5}$

The answer is **A.**

10.

- First use the information from $\cos A = \dfrac{\sqrt{3}}{2},\ 0^0 < \theta < 90^0$ to solve for the angle A.

  Based on the unit circle, A = 30°

- Then use $B = 60^0 + A$ to determine the value of B.

  $B = 60^0 + A$

  $= 60^0 + 30^0$

  $= 90^0$

- Finally, evaluate sec$B$

  $\sec B = \dfrac{1}{\cos B}$

  $= \dfrac{1}{\cos 90^0}$

  $= \dfrac{1}{0}$

  =undefined

The answer is **D**.

NR #2) The length of one complete period is 12 units.
The graph starts at π, ends at 7π, so the length is 6π

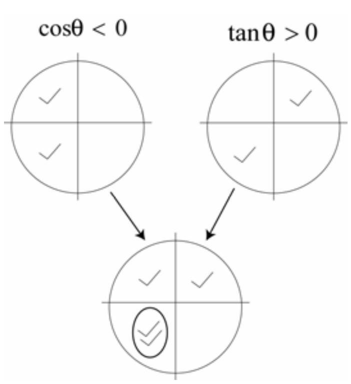
$b = \dfrac{2\pi}{P}$

$= \dfrac{2\pi}{6\pi}$

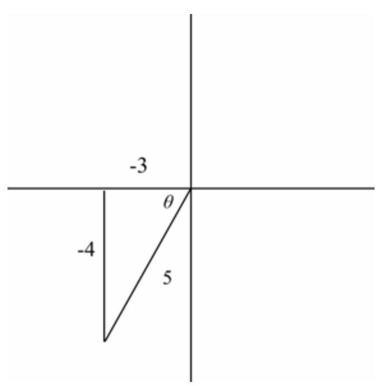
$= \dfrac{1}{3}$

The answer is **0.33**

NR #3) First determine the quadrant the angle can be found in.



Then use Pythagoras to complete the triangle.



$\sin\theta = -\dfrac{4}{5}$

$\cos\theta = -\dfrac{3}{5}$

$\sin^2\theta - \cos^2\theta$

$\left(-\dfrac{4}{5}\right)^2 - \left(-\dfrac{3}{5}\right)^2$

$= 0.28$

The answer is **0.28**

11. The correct answer is C, since the new graph has a phase shift of *c* units to the right. This will move all the θ - intercepts by the same amount. (Remember: In a trig graph, the *x*-intercepts are called θ - intercepts)
The answer is **C.**

12. The only graph which has no effect on the *y*-intercept is $y_3 = \cos 3\theta$, since the horizontal stretch will not transform points on the *y*-axis.
The answer is **B.**

13.

- **amplitude:** $\frac{|\max - \min|}{2} = \frac{|112-28|}{2} = \frac{84}{2} = 42$
- **b-value:** The period is 2π, so $b = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$
- **c-value:** For cosine, it's $\frac{\pi}{2}$
- **d-value:** $\frac{|\max + \min|}{2} = \frac{|112+28|}{2} = \frac{140}{2} = 70$

The equation is $y = 42\cos\left(\theta - \frac{\pi}{2}\right) + 70$

The answer is **C.**

14. First convert to degrees to make the calculation simpler: $\frac{16\pi}{9} \bullet \frac{180^0}{\pi} = 320^0$

The reference angle is 360˚ - 320˚ = 40˚

Now re-convert to radians: $40^0 \bullet \frac{\pi}{180^0} = \frac{2\pi}{9}$

The answer is **A**.

NR #4)
Use the arc length formula to determine the length of the curved portion of the sidewalk.

$a = r\theta$

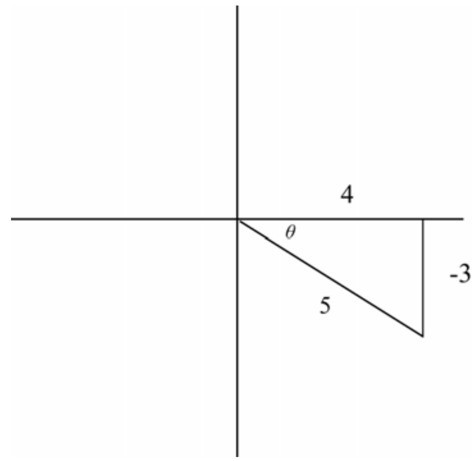
$a = 30 \bullet 1.693$ (*Don't forget to convert to radians!* $97^0 \bullet \frac{\pi}{180^0} = 1.693$)

$a = 51$ m

Add up all the lengths: 30 m + 51 m + 30 m = 111 m

The answer is **111.**

15.
$\frac{3\pi}{2} < \theta < 2\pi$ tells you that the angle is in quadrant IV



Use Pythagoras to determine the remaining side of the triangle

$a^2 + b^2 = c^2$
$4^2 + b^2 = 5^2$
$b^2 = 25 - 16$
$b^2 = 9$
$b = 3$

Thus, $\cot\theta = -\frac{4}{3}$

The answer is **D.**

16. The amplitude of each graph being quadrupled means all the *y*-values are being multiplied by 4.

$\left(\frac{\pi}{8}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{5\pi}{8}, \frac{-\sqrt{2}}{2}\right)$

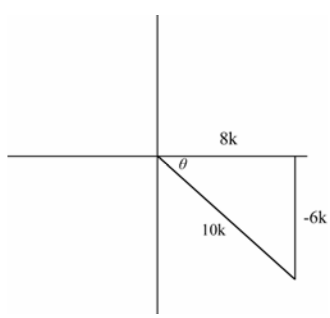
$\left(\frac{\pi}{8},\ 4 \bullet \frac{\sqrt{2}}{2}\right)$, $\left(\frac{5\pi}{8},\ 4 \bullet \frac{-\sqrt{2}}{2}\right)$

$\left(\frac{\pi}{8}, 2\sqrt{2}\right)$, $\left(\frac{5\pi}{8}, -2\sqrt{2}\right)$

The answer is **C.**

17. The values can be represented in a triangle as follows.
Solve for the hypotenuse using Pythagoras.

$a^2 + b^2 = c^2$
$(-6k)^2 + (8k)^2 = c^2$
$36k^2 + 64k^2 = c^2$
$100k^2 = c^2$
$10k = c$



$$\boxed{\begin{aligned} \sin\theta &= \frac{-6k}{10k} = -\frac{3}{5} \\ \cos\theta &= \frac{8k}{10k} = \frac{4}{5} \\ \tan\theta &= \frac{-6k}{8k} = -\frac{3}{4} \end{aligned}}$$

The answer is **B.**

18. $\frac{13\pi}{6}$ is equivalent to $\frac{\pi}{6}$ on the unit circle since they are co-terminal angles.

Evaluate the value of $-3\tan\left(\frac{\pi}{6}\right)$ using the unit circle.

$$-3\tan\left(\frac{\pi}{6}\right)$$

$$-3\frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)}$$

$$-3\bullet\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$-3\bullet\frac{1}{\cancel{2}}\bullet\frac{\cancel{2}}{\sqrt{3}}$$

$$-\frac{3}{\sqrt{3}}$$

Now rationalize the denominator.

$$-\frac{3}{\sqrt{3}}\bullet\frac{\sqrt{3}}{\sqrt{3}}$$

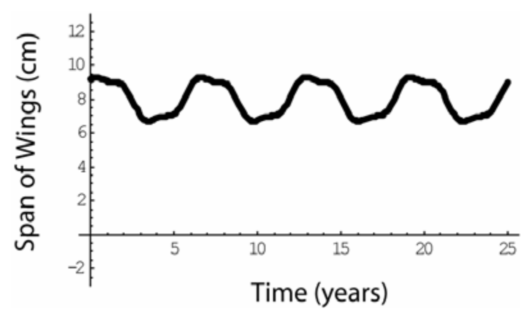
$$\frac{-3\sqrt{3}}{3}$$

$$-\sqrt{3}$$

The answer is **B.**

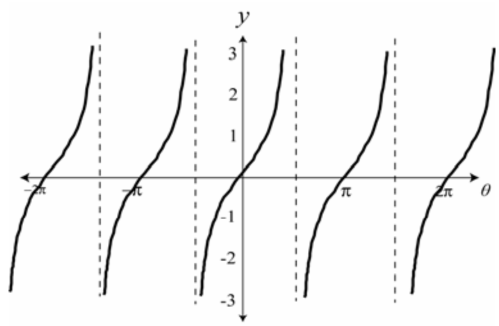
19. Draw the graph of $w(t)=\cos^3 t-\sin(t-3)+8$ using technology in radian mode.

Using the minimum/maximum features of the calculator, the lowest point of a cycle occurs at 6.7 cm, and the highest point at 9.3 cm.



The answer is **B.**

20. Draw in the asymptotes as shown in the diagram to the right. The first asymptote occurs at $\frac{\pi}{2}$, and every other asymptote is a multiple of π units away.



The answer is **D.**

21. To answer this question, determine which of the angles does not reduce to 150˚.

A. -930˚ + 360˚ + 360˚ + 360˚ = 30˚
B. $\frac{17\pi}{6}$ = 510˚ → 510˚ - 360˚ = 150˚
C. $\frac{23\pi}{6}$ = 690˚ → 690˚ - 360˚ = 330˚
D. -3.67 rad = -210˚ → -210˚ + 360˚ = 150˚

Only $\frac{23\pi}{6}$ will give an angle that isn't 30˚ or 150˚
The answer is **C.**

22.

- *a*-value = $\frac{|15-3|}{2}=\frac{|12|}{2}=6\text{m}$

The period is 40 s.

- *b*-value: $\frac{2\pi}{Period}=\frac{2\pi}{40}=\frac{\pi}{20}$
- *c*-value: None
- *d*-value: $\frac{|15+3|}{2}=\frac{|18|}{2}=9\text{m}$

The equation is $h(t)=-6\cos\frac{\pi}{20}t+9$. Use a negative since the cosine graph starts at the bottom of the ride instead of the top.

The answer is **D.**

23. Position B, which is 135° away from the starting position, is $\frac{135^0}{360^0} = 0.375$ revolutions
Multiply this by the period for one complete revolution to find the time.

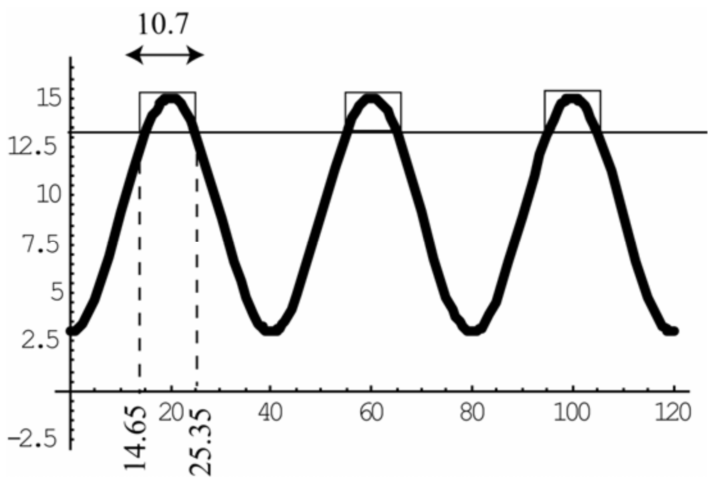
40 s × 0.375 = 15 s.

The answer is **D.**

24. Graph the following:

$y_1 = -6\cos\frac{\pi}{20}t + 9$

$y_2 = 13$



Determine the time a rider spends over 13 m for one cycle, then multiply by 3 to get the complete time for the entire ride.

The total time is 10.7 × 3 = 32 seconds.
The answer is **D.**

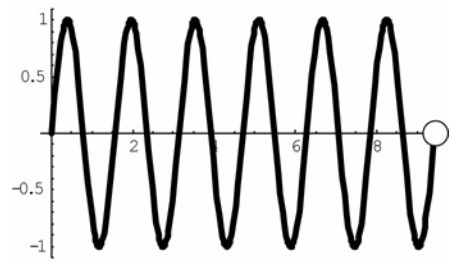
25. When the Ferris Wheel rotates counter-clockwise, there will be **no change** to the shape of the graph, since the height with respect to time will be identical. Look for the answer which will result in exactly the same graph.

The answer which gives the same graph is $y = f(t - 40)$, since moving the cosine shape exactly one full period to the right will have everything re-align.

The answer is **B**.

26. When the ride speeds up, the only thing to be affected is the period. Since the period is related to the *b*-value, the *b*-value will change.
The answer is **B.**

27. The graph is $f(\theta) = \sin 4\theta$ is shown to the right.
There are 12 θ- intercepts between $0 \le \theta < 3\pi$
(Note that you don't include the one at 3π)
So, there are 12 asymptotes.
The answer is **C**.



28.

- a-value = $\frac{|26-20|}{2} = \frac{|6|}{2} = 3$ cm

The period is 2 seconds.

- b-value: $\frac{2\pi}{Period} = \frac{2\pi}{2} = \pi$
- c-value: None
- d-value: $\frac{|20+26|}{2} = \frac{|46|}{2} = 23$ cm

The equation is $h(t) = 3\cos \pi t + 23$.
The answer is **D.**

29. First isolate $\sin\theta$

$$2\sin\theta = \sqrt{3}$$
$$\sin\theta = \frac{\sqrt{3}}{2}$$

Solving from the unit circle, $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$
Each solution repeats for every co-terminal angle, so the general solutions are
$\theta = \frac{\pi}{3} \pm 2n\pi,\ \frac{2\pi}{3} \pm 2n\pi$

The answer is **D**.

30. From the equation, determine the min/max positions, and the length of a cycle.

$$y = 20.1\sin\frac{2\pi}{300}(t - 265) + 6.2 \qquad b = \frac{2\pi}{Period} = \frac{2\pi}{300}$$

Minimum: 6.2 – 20.1 = -13.9
Maximum: 6.2 + 20.1 = 26.3

$$Period = 300$$

A window setting that allows us to see all these points is *x:* [0, 600, 100], *y:* [-15, 30, 5]
The answer is **C**.

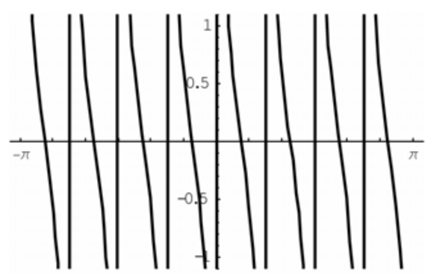
31. First factor the 3 out of the brackets: $g(\theta) = \sin[3\theta - \pi] = \sin 3\left(\theta - \frac{\pi}{3}\right)$

This tells you there is a horizontal stretch by $\frac{1}{3}$, then a phase shift $\frac{\pi}{3}$ units right.

The answer is **D**.

32. The graph of $f(\theta) = \cot 4\theta$ is shown to the right.

The first asymptote occurs along the *y*-axis, and every other asymptote is a multiple of 45˚ away.



The domain is $x \in R,\ x \neq \pm\frac{n\pi}{4}$

The answer is **A**.

Written Response #1

Equation of Sunrise

- a-value = $\dfrac{|10.18-2.57|}{2}=\dfrac{|7.61|}{2}=3.805$

The period is 365 days.

- b-value: $\dfrac{2\pi}{Period}=\dfrac{2\pi}{365}$
- c-value: 10 days left, since the maximum point is on Dec. 21
- d-value: $\dfrac{|10.18+2.57|}{2}=\dfrac{|12.75|}{2}=6.375$

$$T(x)=3.805\cos\frac{2\pi}{365}(x+10)+6.375$$

Equation of Sunset

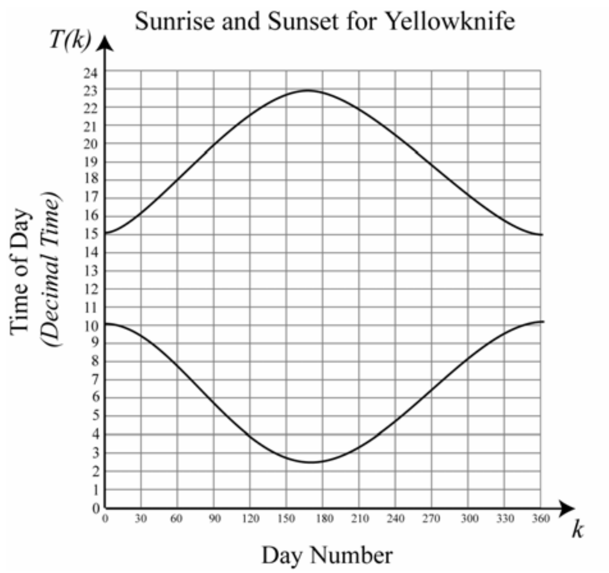
- a-value = $\dfrac{|22.75-15.00|}{2}=\dfrac{|7.75|}{2}=3.875$

The period is 365 days.

- b-value: $\dfrac{2\pi}{Period}=\dfrac{2\pi}{365}$
- c-value: 172 days right, since the maximum point is on June. 21
- d-value: $\dfrac{|22.75+15.00|}{2}=\dfrac{|37.75|}{2}=18.875$

$$T(x)=-3.875\cos\frac{2\pi}{365}(x-172)+18.875$$

Use a negative since the cosine graph is upside-down



The following transformations are required to transform $f(x)=\cos x$ into the graph representing the sunset time in Yellowknife:

Vertical stretch by a factor of 3.875

Reflection in the x-axis.

Horizontal stretch by a factor of $\dfrac{365}{2\pi}$

(Remember to use the reciprocal for a horizontal stretch)

Translation of 172 units right, and 18.875 units up.

Graph $y_1=3.805\cos\dfrac{2\pi}{365}(x+10)+6.375$

$y_2=4$

Determine the intersection points, which occur on days 120 and 225.

Yellowknife experiences a sunrise earlier than 4 AM for 105 days each year.

Feb. 15 is the 46 th day of the year

Graph $y=3.805\cos\dfrac{2\pi}{365}(x+10)+6.375$

2nd → Trace → Value: x = 46. Sunrise = 8.54

Graph $y=3.875\cos\dfrac{2\pi}{365}(x-172)+18.875$

2nd → Trace → Value: x = 46. Sunset = 16.69

Length of daylight = 16.69 – 8.54 = 8.15 hours.

(Can also be written as 8 hours, 9 minutes.)

Written Response #2

- If it takes 1 second for the tire to go around twice, that means the time for one spin is 0.5 seconds. *The period is 0.5 s.*
-

Parameter	Value
a	42
b	12.57
c	0
d	42

- $h(t) = -42\sin 12.57t + 42$
- Contact between the wheel and ground occurs when the height, $h(t)$, is zero.

Solve the equation: $0 = -42\sin 12.57t + 42$ by graphing to obtain the *t*-intercepts.

The first contact occurs at 0.125 seconds, and every future contact occurs one period later. The general formula is: *Time of contact = 0.125 + 0.5n*

The fifth contact will be 0.125 + 0.5(4) = **2.125 s**

- *a*-value: 39 instead of 42

Period: Still 0.5 seconds

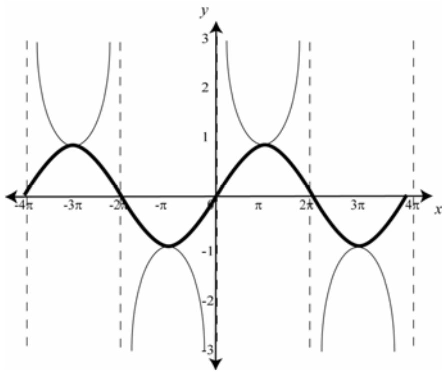
b-value: No change, since no change in period

c-value: No change

d-value: Midline is at 39 instead of 42.

Written Response #3

a - value	1
b - value	$\frac{1}{2}$
Phase Shift	0
Vertical Displacement	0
Period	4π
Domain	$x \neq \pm 2n\pi$
Range	$y \leq -1,\ y \geq 1$
x-intercepts	None
y-intercepts	None
Asymptotes (general equation)	$x = \pm 2n\pi$



- The equation of the graph is $g(\theta) = \sin\frac{1}{2}\theta$
- The asymptotes in $f(\theta)$ occur at the θ- intercepts of $g(\theta)$

$$f\left(\frac{10\pi}{3}\right) = \csc\frac{1}{2}\left(\frac{10\pi}{3}\right)$$
$$= \csc\left(\frac{10\pi}{6}\right)$$
$$= \csc\left(\frac{5\pi}{3}\right)$$

$$= \frac{1}{\sin\left(\frac{5\pi}{3}\right)}$$
$$= \frac{1}{-\frac{\sqrt{3}}{2}}$$
$$= 1 \times -\frac{2}{\sqrt{3}}$$
$$= -\frac{2}{\sqrt{3}}$$

Now rationalize the denominator:

$$= -\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= -\frac{2\sqrt{3}}{3}$$