Principles of Mathematics 12

Trigonometry II Practice Exam

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Trigonometry II Practice Exam

Use this sheet to record your answers

1.	NR 2.	19.	28.
2.	NR 3.	20.	NR 7.
NR 1.	11.	21.	29.
3.	12.	NR 5.	30.
4.	13.	22.	31.
5.	14.	23.	32.
6.	NR 4.	24.	33.
7.	15.	25.	
8.	16.	NR 6.	
9.	17.	26.	
10.	18.	27.	

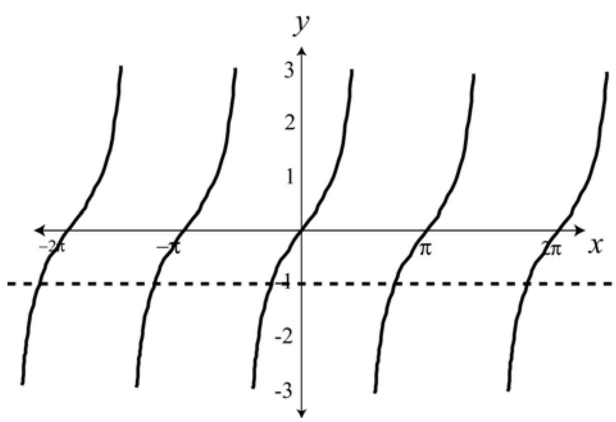
Trigonometry II Practice Exam

1. The exact value of sin 75˚ can be determined using the expression

A. sin 90˚ - sin 15˚
B. sin 45˚ + sin 30˚
C. sin 45˚cos 30˚ + cos 45˚sin 30˚
D. cos 45˚cos 30˚ + cos 45˚cos 30˚

Use the following information to answer the next question.

A student solves the equation $\tan x = -1$ in their graphing calculator as shown in the diagram below.



The student determines the general solution of this graph is $-\frac{\pi}{4} + n\pi,\ n \in I$

2. The general solution to the equation $\tan(5x) = -1$ is

A. $-\frac{\pi}{3} + \frac{n\pi}{10},\ n \in I$

B. $-\frac{\pi}{4} + n\pi,\ n \in I$

C. $-\frac{5\pi}{4} + 5n\pi,\ n \in I$

D. $-\frac{\pi}{20} + \frac{n\pi}{5},\ n \in I$

Numerical Response

1. The identity $\cot^2 x + \csc x = \dfrac{\cos^2 x + \sin x}{\sin^2 x}$ may be verified by substituting 2.1 rad for x on each side. When this substitution is made, the numerical value of each side, to the nearest hundredth, is __________.

Use the following information to answer the next question.

A Ferris wheel at an amusement park, with a diameter of 12 m, can be modeled using the equation $h(t) = -6\cos\dfrac{\pi}{20}t + 9$, where $h(t)$ is the height above the ground in metres, and t is the time in seconds.

3. The number of seconds required for a rider to reach a height of 14 m for the first time is, to the nearest tenth,

A. 16.3 s
B. 16.5 s
C. 20.4 s
D. 932 s

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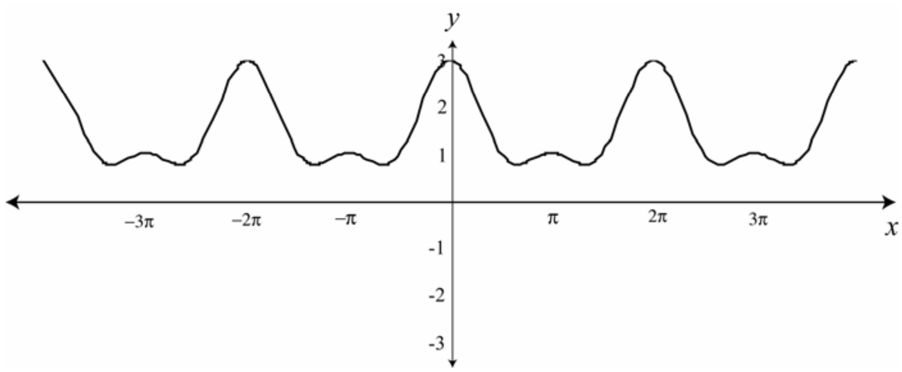
4. The expression $\cot^2 x + \csc x - 4$ is equivalent to

A. $\csc^2 x + \csc x - 5$
B. $\csc^2 x + \csc x - 3$
C. $\dfrac{\cos^2 x}{\sin^2 x} + \dfrac{1}{\cos x} - 4$
D. -4

Use the following information to answer the next question.

The graph of $f(x) = \cos^2 x + \cos x + 1$ is shown below



5. If the equation $\cos^2 x + \cos x + 1 = 3$ has the general solution $2n\pi,\ n \in I$, then a possible solution to the equation $\cos^2\left(\frac{x}{3}\right) + \cos\left(\frac{x}{3}\right) + 1 = 3$ is

A. 2π
B. 3π
C. 9π
D. 12π

6. If $\sin A = \frac{m}{n}$ and $\tan A = \frac{m^2}{n^3},$ where $m, n \neq 0,$ then $\cos A$ is equivalent to

A. $\frac{n^2}{m}$
B. $\frac{m^3}{n^4}$
C. mn^2
D. $\frac{1}{mn^2}$

7. The expression $\sqrt{\frac{1+\tan^2 x}{1-\sin^2 x}}$, is equivalent to

A. $\sqrt{\frac{(1+\tan x)(1-\tan x)}{(1+\sin x)(1-\sin x)}}$
B. 1
C. $\sec x$
D. $\sec^2 x$

8. If $\tan^2 x = \frac{5}{7}$, then $\sec^2 x$ is equivalent to

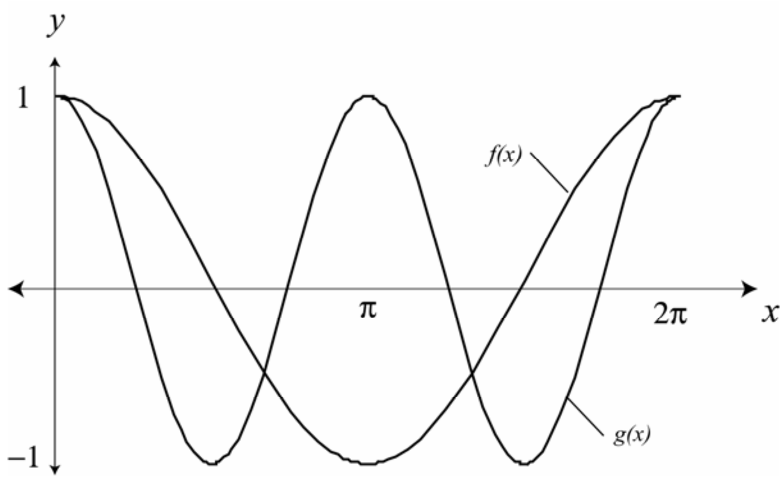
A. $\frac{12}{7}$
B. $\frac{7}{5}$
C. $\frac{5\sqrt{74}}{74}$
D. $\frac{\sqrt{74}}{7}$

9. The expression $\cos^2(4\pi)-\sin^2(4\pi)$ is equivalent to

A. $\cos^2(4\pi)$
B. $\sin^2(8\pi)$
C. $\cos(8\pi)$
D. $\cos(4\pi)\sin(4\pi)$

Use the following information to answer the next question.

The graphs of $f(x) = \cos x$ and $g(x) = \cos(2x)$ are shown below



The graphs intersect four times on the interval $0 \le x \le 2\pi$

10. If the domain is changed to $0 < x < 2\pi$, (*the equality has been removed*) a correct statement is

A. There are more solutions
B. There are fewer solutions
C. There are the same number of solutions
D. There is no change in the number of solutions

Numerical Response

2. The equation $\csc^2 x - 2 = \cos^2 x$ has four solutions in the interval $0 < x < 2\pi$. The number of solutions for x in the interval $0 < x < 14\pi$ is _________.

Use the following information to answer the next question.

The steps used by a student to simplify the expression $(\sin x + \cos x)^2$ are shown below

$$\textbf{Step 1:}\quad \sin^2 x + \cos^2 x$$
$$\textbf{Step 2:}\quad \sin^2 x + \left(1 - \sin^2 x\right)$$
$$\textbf{Step 3:}\quad \left(1 - \cos^2 x\right) + \left(1 - \sin^2 x\right)$$
$$\textbf{Step 4:}\quad 2 - \sin^2 x - \cos^2 x$$

Numerical Response

3. The step which contains a mathematical error is step ________.

11. The value of m in the equation $\dfrac{m \sin x \cot x}{4 \csc x \tan x} = 8$ is

A. $\dfrac{\sin x \cos x}{32}$
B. $\dfrac{2 \sin x \cos x}{\tan x}$
C. $32 \sec^2 x$
D. $32 \sec x \csc x$

12. The solutions to the equation $\cos^2 x = \cos x$, where $0 \le x < 2\pi$ are

A. $0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi$
B. $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$
C. $0, \dfrac{\pi}{2}, \dfrac{3\pi}{2}$
D. $0, \dfrac{\pi}{2}, \dfrac{3\pi}{2}, 2\pi$

13. The expression $\frac{\cos x}{1-2\sin x}$ is undefined when the values of x are

A. $\frac{\pi}{6}\pm 2n\pi,\ \frac{5\pi}{6}\pm 2n\pi$

B. $\frac{\pi}{6}\pm n\pi$

C. $\frac{\pi}{6}\pm 2n\pi\ ,\ \frac{5\pi}{6}\pm 2n\pi\ ,\ \frac{\pi}{2}\pm n\pi$

D. $\frac{n\pi}{2}$

14. The expression $\sec\left(x-\frac{\pi}{2}\right)$ is equivalent to

A. $\sec x-\sec\frac{\pi}{2}$

B. $\cos\left(x-\frac{\pi}{2}\right)$

C. $\csc x$

D. $-\sin\left(\frac{\pi}{2}-x\right)$

Numerical Response

4. If $\frac{1}{1+\cot^2 x}=0.43$, and $0\leq x<\frac{\pi}{2}$, then the value of x in radians, to the nearest tenth, is ________.

15. The expression $\frac{\sin x}{\tan x}+\frac{1}{\sec x}$ is equivalent to

A. $2\cos x$

B. $2\sec x$

C. $\frac{\sin x+1}{\tan x+\sec x}$

D. $\frac{\sin x}{\tan x\sec x}$

16. Given $\sin A = \frac{7}{8}$ and $\cos B = \frac{4}{5}$, where A and B are acute angles, the value of $\cos(A - B)$ is equal to

A. $\frac{3}{8}$

B. $\frac{16}{25} + \frac{49}{64}$

C. $\frac{4\sqrt{15} + 21}{40}$

D. $\frac{28 - 3\sqrt{15}}{40}$

17. If the equation $-5\csc^2 x + 12\cot^2 x - 9 = 0$ is simplified using the identity $1 + \cot^2 x = \csc^2 x$, the resulting equation is

A. $-5\tan^2 x + 12\sec^2 x - 9 = 0$

B. $\cot^2 x = 2$

C. $12\cot^2 x - 9 - 5\sec^2 x = 0$

D. $\sec 2x\left(1 - \tan^2 x\right) = 6$

18. The expression $\cos(x - y) - \cos(x + y)$ is equivalent to

A. $2\sin x \sin y$

B. 0

C. $-2\cos y$

D. $\cos\left(\frac{x - y}{x + y}\right)$

19. The line $y = \frac{1}{2}$ intersects the graph of $\cos^2 x - \sin x$ twice in the interval $0 \le x < 2\pi$. An equation that can be used to solve for x is

A. $\cos^2 x = \sin x$

B. $2\cos^2 x - 2\sin x - 1 = 0$

C. $\sin x - \cos^2 x = 2$

D. $2\cos^2 x + 2\sin x - 1 = 0$

20. The expression $\sin\left(\frac{\theta}{5}\right)\cos\left(\frac{2\theta}{7}\right)-\cos\left(\frac{\theta}{5}\right)\sin\left(\frac{2\theta}{7}\right)$ is equivalent to

A. $\cos\left(\frac{17\theta}{35}\right)$

B. $\sin\left(\frac{17\theta}{35}\right)$

C. $\sin\left(\frac{3\theta}{35}\right)$

D. $\sin\left(\frac{-3\theta}{35}\right)$

21. If $\frac{\csc 2x}{\sec 2x}=\sqrt{5}$, then the value of *x*, to the nearest hundredth of a radian is

A. $1.15+3.14n\ ,\ n\in I$
B. $0.42+3.14n\ ,\ n\in I$
C. $0.54+1.57n\ ,\ n\in I$
D. $0.21+1.57n\ ,\ n\in I$

Use the following information to answer the next question.

A student is given four different trigonometric expressions

I	$\frac{1}{9}\sec x\cos x$
II	$\cot^2 x-\csc^2 x$
III	$2\cos^2 x+2\sin^2 x$
IV	$2\cot x - \frac{2\cos x}{\sin x}$

Numerical Response

5. If the expressions are simplified are ranked, from smallest to largest, the correct order is ________.

22. Given $\sin x = m$, an expression for $\cos 2x$, in terms of *m*, is

A. $1-2m^2$
B. $1-2m$
C. $2m^2-1$
D. $2m-1$

23. The expression $\dfrac{1+\csc x}{\sin x}$ is equivalent to

A. $\csc x+\sin x$
B. $\dfrac{\sin x+1}{\cos^2 x+1}$
C. $\dfrac{\sin x+1}{\sin^2 x}$
D. 1

24. Given $x = 45^0$, an equivalent expression to $\dfrac{\cos(x+y)}{\cos y}$ is

A. $\cos\left(\dfrac{x}{y}\right)+1$
B. $\dfrac{\sqrt{2}}{2}(1-\tan y)$
C. $\cos x$
D. $\dfrac{2+\sqrt{2}\cos y}{2\cos y}$

25. The exact value of $\sec\left(-\dfrac{\pi}{12}\right)$ is

A. $\dfrac{4}{\sqrt{2}-\sqrt{6}}$
B. $\sqrt{6}-\sqrt{2}$
C. -75^0
D. $\dfrac{11\pi}{12}$

Numerical Response

6. The expression $\cos x$ may be written as $\cos^2 kx - \sin^2 kx$. The value of *k*, to the nearest tenth, is ________.

26. Using the identity $\cos^2 x = 1 - \sin^2 x$, the expression $\cos^2 x - \sin^2 x - 1 + 2\sin x$ can be simplified to

A. $2\sin x(1-\sin x)$
B. $\sin x(1-2\sin x)$
C. $\sin 2x + 2\sin x$
D. $2\sin 2x + 1$

27. If $\tan x = -\frac{6}{7}$ and $\sin y = -\frac{2}{5}$, the exact value of $\sec(x+y)$, given that $\frac{3\pi}{2} \le x < 2\pi$, $\frac{3\pi}{2} \le y < 2\pi$, is

A. $\sqrt{21} - 5$
B. $5 - \sqrt{21}$
C. $\frac{7\sqrt{21}}{12\sqrt{85} - 5}$
D. $\frac{5\sqrt{85}}{7\sqrt{21} - 12}$

28. The expression $\csc x - \sin x$ is equivalent to

A. $\frac{1}{\sin^2 x}$
B. 1
C. $\frac{\sin x}{\cos^2 x}$
D. $\cot x \cos x$

Numerical Response

7. The number of solutions in the equation $\tan^2 x = 1$, where $0 \le x < 2\pi$, is __________.

29. The expression $\dfrac{\sin x + \tan x}{\cos x + 1}$ is equivalent to

A. $\csc^2 x$
B. $\tan x$
C. $\dfrac{2\sin x}{\cos x + 1}$
D. $2\tan x$

30. The expression $\csc^4 x - 1$ is equivalent to

A. $\dfrac{\csc^4 x}{\sec^4 x}$
B. $\cot^4 x$
C. $\cot^2 x\left(\csc^2 x + 1\right)$
D. $\cot^2 x\left(\sec^2 x + 1\right)$

31. The general solution to the equation $\sin 4x = -\dfrac{1}{2}$ is

A. $\dfrac{7\pi}{24} \pm \dfrac{n\pi}{2}$
B. $\dfrac{5\pi}{12} \pm \dfrac{n\pi}{4},\ \dfrac{3\pi}{12} \pm \dfrac{n\pi}{4}$
C. $\dfrac{7\pi}{24} \pm \dfrac{n\pi}{2},\ \dfrac{11\pi}{24} \pm \dfrac{n\pi}{2}$
D. $\dfrac{3\pi}{12} \pm \dfrac{n\pi}{4}$

32. The expression $\sec 2x$ is undefined when x is the angle

A. $\frac{\pi}{4}$
B. $\frac{\pi}{2}$
C. π
D. 2π

Use the following information to answer the next question.

A student solves the equation $\cos^2 x - 2 = 0$ algebraically, using the steps shown below

$$\left(\cos x - \sqrt{2}\right)\left(\cos x + \sqrt{2}\right) = 0$$

$\cos x - \sqrt{2} = 0 \quad \rightarrow \quad x$ has no solution.

$\cos x + \sqrt{2} = 0 \quad \rightarrow \quad x$ has no solution.

33. The reason why $\cos^2 x - 2 = 0$ has no solution is because

A. $\cos x$ is undefined for $x = \sqrt{2} \pm 2n\pi$
B. The range of $y = \cos x$ is $-1 \le y \le 1$
C. $\cos^2 x - 2 = 0$ cannot be factored
D. $\cos^2 x$ must be replaced with $\sin^2 x - 1$ before factoring

Use the following information to answer the next question.

A student graphs the following function in a graphing calculator.

$$f(x) = 8 - 3\sin^2 x$$

x is measured in radians, and the student wishes to analyze the graph for $-2\pi \le x \le 2\pi$

Written Response – 10%

1.

- Explain how the student would have to type the above equation into their graphing calculator in order to obtain the correct graph. Indicate appropriate window settings.

- The student now wishes to solve the equation $6.2 = f(x)$.
  State the general solution to this equation in radian decimal form, to the nearest hundredth.

- The graph of $f(x)$ can be expressed in the form $g(x) = a\cos b[x - c] + d$. Write the equation for $g(x)$

- Algebraically solve the equation $7 + \sin^2 x = 8 - 3\sin^2 x$
  Show all steps required in obtaining the answer.

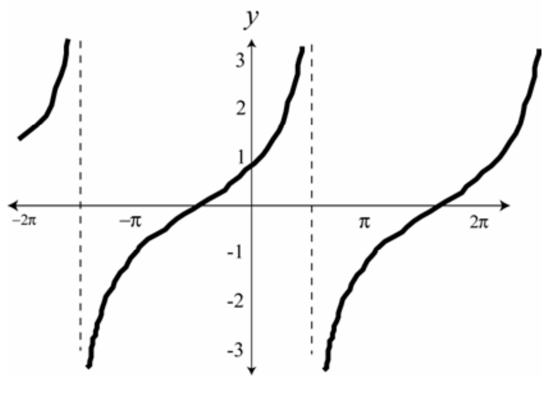
Written Response – 10%

2.

- Verify the identity $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$ for $x = \frac{\pi}{6}$

Use the following additional information to answer the next part of the question.

The graphs of $y_1 = \frac{\cos x}{1-\sin x}$ and $y_2 = \frac{1+\sin x}{\cos x}$ are shown below.



- The graphs of $y_1 = \frac{\cos x}{1-\sin x}$ and $y_2 = \frac{1+\sin x}{\cos x}$ are **not** identical. Explain the difference between the graphs of y_1 and y_2.

- Algebraically prove the identity $\frac{\cos x}{1-\sin x}=\frac{1+\sin x}{\cos x}$

- Algebraically show that $\frac{\cos x}{1-\sin x}+\frac{1+\sin x}{\cos x}=\frac{2\cos x}{1-\sin x}$

Written Response – 10%

3.

- Prove the identity $\dfrac{1+\cos 2x}{\sin 2x}=\cot x$

- Prove the identity $(\sin x+\cos x)^2=1+\sin 2x$

- Prove the identity $\sin 2x=2\sin x\cos x$

- Solve algebraically: $2\sin x \cos x = \cos x$

- Solve algebraically: $\frac{\sin x}{2} = \frac{\sin x}{3}$

- Solve algebraically: $\frac{\csc x}{5} + \frac{\csc x}{3} = \frac{16}{15}$