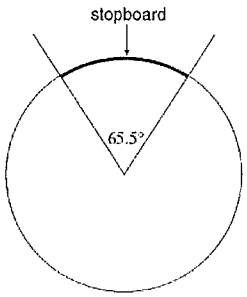
Pre-Calculus 12 Practice Exam 1
MULTIPLE-CHOICE (Calculator permitted)

1. Determine the smallest zero for $y = 4\sin 3\theta + 2$ in the interval $2\pi \le \theta \le 3\pi$

   a. 0.38 b. 1.22 c. 6.66 d. 7.50

2. In high school, a shot put is thrown out of a circle with a radius of 3.5 feet. A curved wooden "stopboard" is placed in an arc around part of this circle. The central angle is 65.5°. Determine the length of the curved "stopboard."



   a. 3.5 ft
   b. 4.0 ft
   c. 4.5 ft
   d. 5.0 ft

3. On Oct. 2, 2010, the tide at New Westminster reached a maximum height of 10.8 feet at midnight. At 9 am the tide reached the next minimum height of 5.8 feet. Assuming the relationship is sinusoidal, what was the height of the tide at 7 am?

   a. 6.1 ft b. 6.4 ft c. 8.7 ft d. 9.9 ft

4. Solve $7 = 2^{x+1}$.

   a. -0.64 b. 1.36 c. 1.81 d. 3.81

5. The population in a particular community is increasing at an annual rate of 6.5%. Assume this trend will continue. In how many years will the present population of 12 000 grow to 32 000?

   a. 15.5 b. 15.6 c. 15.8 d. 16.1

6. In a study which compared the pH of urine and tears, the following data was collected.

	Urine	Tears
Joe	6.2	7.6
Bob	6.3	7.4
Bill	5.5	7.5
Average	**6.0**	**7.5**

   On average, how many times more alkaline are tears than urine?

   a. 1.3 c. 15.0
   b. 1.5 d. 31.6

7. A restaurant offers a selection of 4 different sandwiches, 3 different soups and 4 different flavours of juice. In how many different ways can a person select one item from each category?

   a. 11 b. 18 c. 48 d. 54

8. Determine the 4th term in the expansion of $\left(4x - \frac{y}{2}\right)^8$

   a. $-7168x^5y^3$ c. $16x^4y^4$
   b. $-128x^5y^3$ d. $1120x^4y^4$

9. A family of 6 (2 parents and 4 children) sit in a row at a theatre. A parent must sit at either end with the 4 children between them. In how many ways can the family be seated?

   a. 24 b. 48 c. 120 d. 720

10. From a standard deck of 52 cards, how many 5-card hands can be formed containing at least 4 clubs?

    a. 20 592 b. 27 885 c. 29 172 d. 34 320

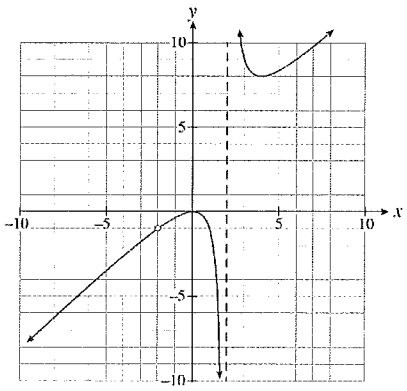
11. A hockey team has played 10 games and has a record of 5 wins, 3 losses and 2 ties. In how many ways could this have happened if after the first 4 games the team's record was 3 wins and a loss?

    a. 90 b. 94 c. 360 d. 630

12. When the polynomial $P(x)$ is divided by $x+3$, the remainder is 2. Which point must be on the graph corresponding to the function $y = P(x)$?

    a. $(-3,-2)$ b. $(-3,0)$ c. $(-3,2)$ d. $(3,2)$

13. Determine the range of the rational function graphed below.



    a. all reals
    b. $(-\infty,0],[8,\infty)$
    c. $(-\infty,2],(2,\infty)$
    d. $(-\infty,-1),(-1,0],[8,\infty)$

14. The graph $y=f(x)$ is stretched horizontally by a factor of $\frac{1}{4}$. Determine the equation of the transformed graph.

a. $y=\frac{1}{4}f(x)$ c. $y=f(\frac{1}{4}x)$

b. $y=4f(x)$ d. $y=f(4x)$

MULTIPLE-CHOICE (Calculator NOT permitted)

15. The students at a graduation dinner are separated into groups to be seated at 10 different tables. The order in which these 10 groups will approach the buffet is to be determined randomly. In how many ways can this order be determined?

a. 10^{10} b. ${}_{10}C_{10}$ c. ${}_{10}P_{10}$ d. 10×10

16. Consider the graph of $y=-3\cos\frac{\pi(x-2)}{10}+4$.

Which statement is false?

I.	The amplitude is 3
II.	The period is 10
III.	The phase shift is 2 to the right
IV.	The vertical displacement is 4 up

a. I
b. II
c. III
d. IV

17. Solve $\cos x=\frac{\sqrt{3}}{2}$, $0\le x<2\pi$

a. $\frac{\pi}{6},\frac{5\pi}{6}$ b. $\frac{\pi}{6},\frac{11\pi}{6}$ c. $\frac{\pi}{3},\frac{2\pi}{3}$ d. $\frac{\pi}{3},\frac{5\pi}{3}$

18. Determine an expression for all angles coterminal with a standard position angle measuring 120°. Express your answer in radians.

a. $\frac{5\pi}{6}+\pi n, n\in Z$ c. $\frac{5\pi}{6}+2\pi n, n\in Z$

b. $\frac{2\pi}{3}+\pi n, n\in Z$ d. $\frac{2\pi}{3}+2\pi n, n\in Z$

19. Determine the exact value of $\tan75°$

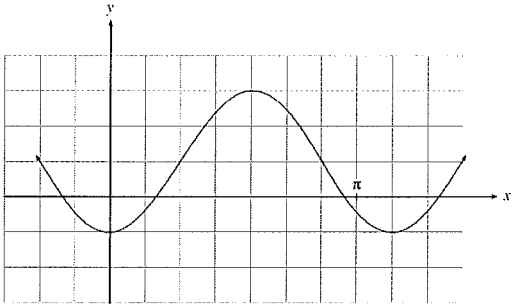
a. $2+\sqrt{3}$ c. $\frac{5+\sqrt{3}}{4}$

b. $-2-\sqrt{3}$ d. $\frac{3+\sqrt{3}}{\sqrt{3}}$

20. A point with an x value of 2 lies on the circle with equation $x^2+y^2=5$. This point also lies on the terminal arm of θ in standard position. Determine the value of $\sec\theta$.

a. $\frac{\sqrt{5}}{2}$ b. $\frac{5}{2}$ c. $\frac{2}{\sqrt{5}}$ d. $\frac{2}{5}$

21. The graph of $y=2\sin b(x-c)+1$ is shown below. Determine a value of c.



a. $-\frac{2\pi}{2}$ b. 2 c. $\frac{\pi}{4}$ d. $\frac{2\pi}{7}$

22. Determine all restrictions for the expression $\frac{\tan x}{\cos x-1}$

a. $\cos x\neq0$ c. $\sin x\neq0, \cos x\neq1$

b. $\cos x\neq1$ d. $\cos x\neq0, \cos x\neq1$

23. Solve $\sin x=-\cos x$, $-\pi\le x\le\pi$

a. $-\frac{\pi}{4},\frac{3\pi}{4}$ b. $\frac{\pi}{4},\frac{3\pi}{4}$ c. $\frac{3\pi}{4},\frac{7\pi}{4}$ d. $\frac{3\pi}{4},\frac{5\pi}{4}$

24. Simplify $\frac{\csc\theta-\sin\theta}{\sec\theta-\cos\theta}$

a. $\cot^2\theta$ b. $\cot^3\theta$ c. $\tan^2\theta$ d. $\tan^3\theta$

25. Solve $2^{3x-1}=8^{2x+1}$

a. $x=-\frac{4}{3}$ b. $x=-1$ c. $x=-\frac{2}{3}$ d. $x=-\frac{3}{4}$

26. Express $\log\frac{x^2}{10y^3}$ in terms of $\log x$ or $\log y$

a. $2\log x-1-3\log y$ c. $2\log x-10-3\log y$

b. $2\log x-1+3\log y$ d. $2\log x-10+3\log y$

27. Evaluate $\log_3 \sqrt{27}$

a. $\frac{2}{9}$ b. $\frac{2}{3}$ c. $\frac{3}{2}$ d. $\frac{9}{2}$

28. Bart and Arnie presented separate solutions to the statement "Write $\log_2 x + \log_4 y$ as a single log"

Bart	Arnie
$\log_2 x + \log_4 y$	$\log_2 x + \log_4 y$
$= \frac{\log_4 x}{\log_4 2} + \log_4 y$	$= \log_2 x + \frac{\log_2 y}{\log_2 4}$
$= 2\log_4 x + \log_4 y$	$= \log_2 x + \frac{1}{2}\log_2 y$
$= \log_4 x^2 y$	$= \log_2 x\sqrt{y}$

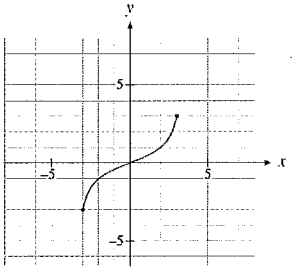
Which statement is true?

a. Only Bart is correct
b. Only Arnie is correct
c. They are both wrong
d. They are both correct

29. Which statement must be true for $f(x) = \log_{\frac{1}{2}} x$ when $x_2 > x_1$?

a. $f(x_1) > f(x_2)$
b. $f(x_2) > f(x_1)$
c. $f(x_1) > 0, f(x_2) < 0$
d. $f(x_2) > 0, f(x_1) < 0$

30. For which graph is the relation and its inverse both functions?

a.


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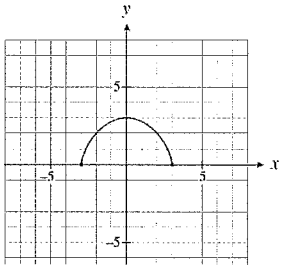

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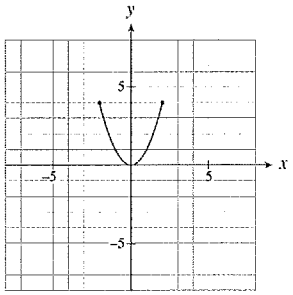

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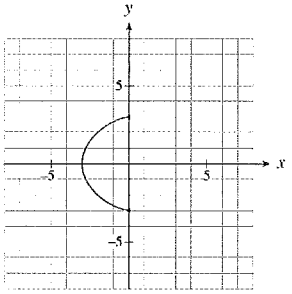
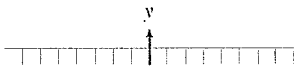
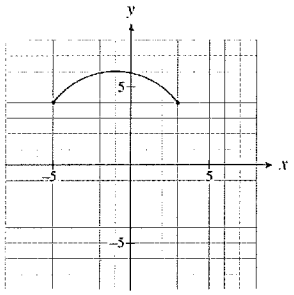


31. The graph of $y = f(x)$ is shown below.



Which graph represents the graph of $y = f(2(x-3))+4$?

a.


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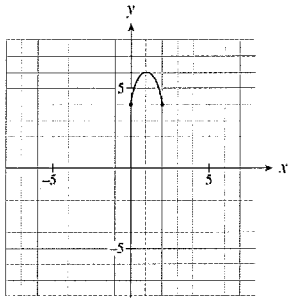

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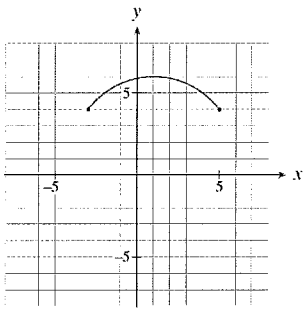

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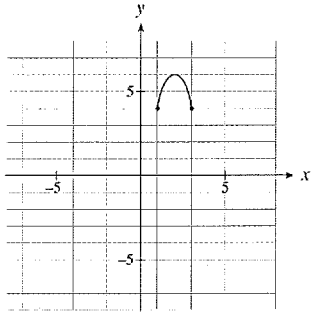


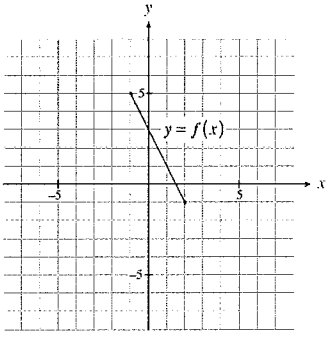
32. Consider the following transformations on the graph of $y = f(x)$

I.	$y = f(x+2)$
II.	$y = 2f(x)$
III.	$y = f(-x)$
IV.	$y = -f(x)$

Which transformations will have no effect on the zeros of the original graph of $y = f(x)$?

a. I and II only
b. II and III only
c. II and IV only
d. III and IV only

33. The graph of $y = f(x)$ as shown below is transformed to $x = f(y)$. Determine all invariant points.



a. $(0,3)$
b. $(1,1)$
c. $(2,-1)$
d. $(1,1)$ and $(2,-1)$

34. The point $P(4,6)$ lies on the graph of $y=f(x)$. Which point must lie on the graph $y=-\frac{1}{2}f\left(\frac{1}{2}x+2\right)$?

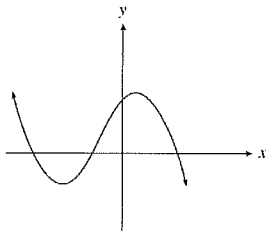
a. $(7,-3)$ b. $(4,-3)$ c. $(1,-3)$ d. $(-2,-3)$

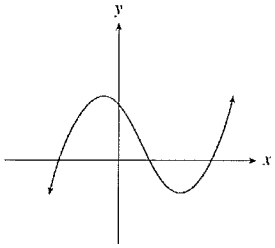
35. Which of the following functions are polynomial functions?

I.	$y=x^3-\sqrt{2}x^2+x+3$
II.	$y=x^3-\frac{2}{x^2}-x+3$
III.	$y=x^3-2x^{1.5}+x+3$
IV.	$y=x^3-\frac{1}{2}x^2-x+3$

a. III only
b. IV only
c. I and IV only
d. II and III only

36. Which sketch best represents the graph of $y=ax^3-bx^2+cx+24$ if $a<0$?

a.


c.


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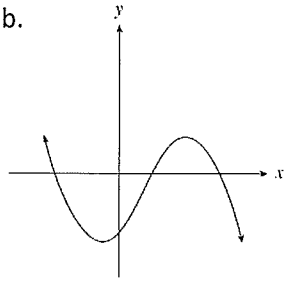
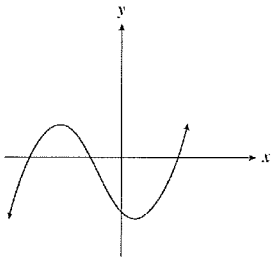


d.


37. Which three expressions are factors of $y=9x^3-36x^2-4x+16$?

I.	$x-4$
II.	$x+4$
III.	$3x-2$
IV.	$3x+2$

a. I, II, III only
b. I, II, IV only
c. I, III, IV only
d. II, III, IV only

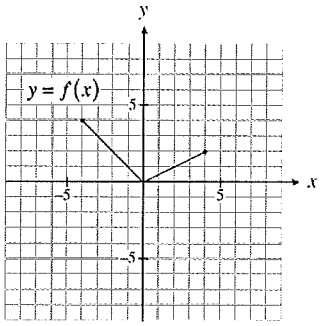
38. When $x^3-2kx^2+3k^2x-15$ is divided by $x-2$, the remainder is 1. Determine all values of k.

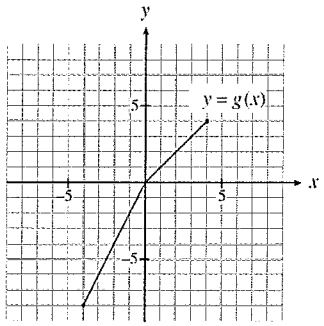
a. $k=-4$ b. $k=\frac{17}{8}$ c. $k=-\frac{2}{3},2$ d. $k=\frac{2}{3},-2$

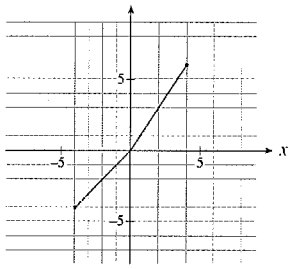
39. Given $f(x)=x+2$ and $g(x)=x^2+3x-1$, determine the value of $f(g(3))$.

a. 16 b. 17 c. 19 d. 39

40. The graphs of $y=f(x)$ and $y=g(x)$ are shown below. Which graph represents $y=f(x)+g(x)$?





a.


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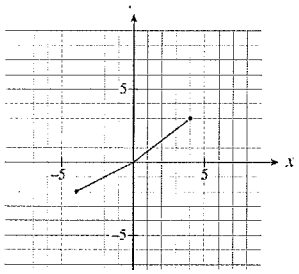

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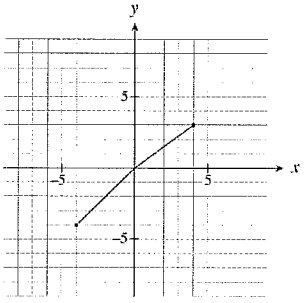

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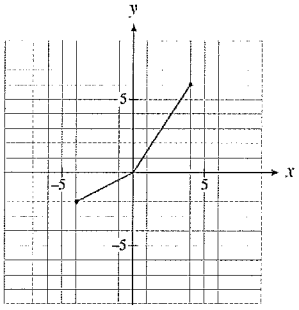


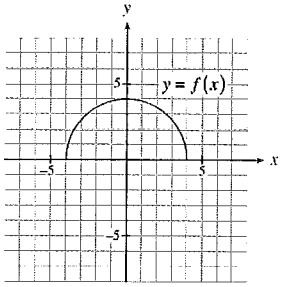
41. As a furniture salesperson, Chacci gets a 3% commission on all his weekly sales above $5000. Which row in the table shows the composite function that will determine his commission if x represents his weekly sales?

	Amount Eligible for Commission $f(x)$	Commission $g(x)$	Composite function
a.	$f(x)=5000-x$	$g(x)=0.03x$	$f(g(x))$
b.	$f(x)=x-5000$	$g(x)=0.03x$	$f(g(x))$
c.	$f(x)=5000-x$	$g(x)=0.03x$	$g(f(x))$
d.	$f(x)=x-5000$	$g(x)=0.03x$	$g(f(x))$

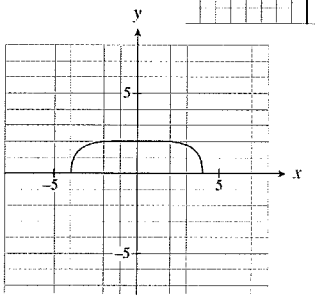
42. Consider the graphs of the functions $f(x)=x^2$ and $g(x)=\sqrt{f(x)}$. Which row describes the domain and range of $g(x)$?

	Domain	Range
a.	all reals	all reals
b.	has restrictions	has restrictions
c.	has restrictions	all reals
d.	all reals	has restrictions

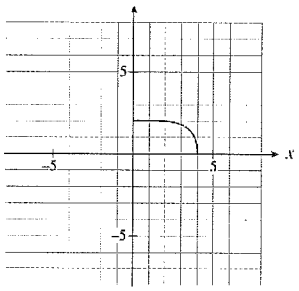
43. Given the graph of $y=f(x)$ as shown, determine the graph of $y=\sqrt{f(x)}$



a.



c.



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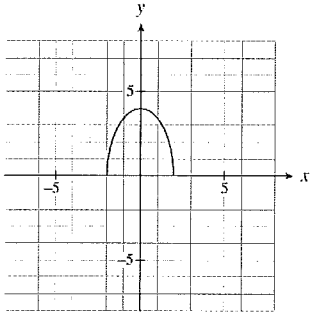

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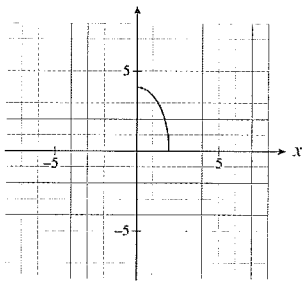


44. Determine the range of the function $y=\sqrt{3x-9}+2$

a. $y\geq 0$ b. $y\geq 2$ c. $y\geq 3$ d. $y\geq 9$

WRITTEN-RESPONSE QUESTIONS (Calculator permitted)

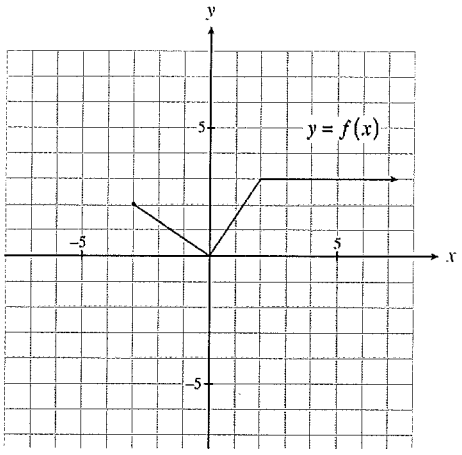
1. Solve algebraically, over the set of real numbers, giving exact values where possible. **(4 marks)**

$$3\sin^2\theta+5\cos\theta=1$$

2. Prove algebraically. **(4 marks)**

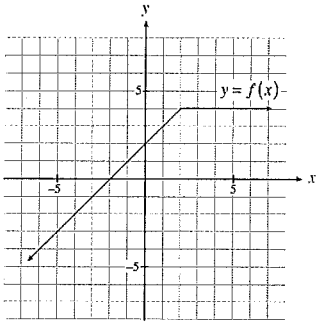
$$\frac{\tan 2\theta(1-\tan\theta)\cos^2\theta}{\sin 2\theta}=\frac{1}{1+\tan\theta}$$

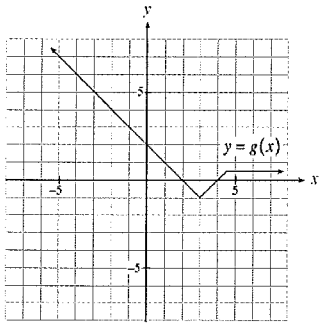
3. The graph of $y=f(x)$ is shown below.



Sketch the graph of $y=2f(-x)-3$ **(4 marks)**

4. Use the graphs of $y=f(x)$ and $y=g(x)$ shown to sketch the graph of $y=f(x)g(x)$. Label enough points to get an accurate representation of the graph. **(4 marks)**





WRITTEN-RESPONSE QUESTIONS (Calculator NOT permitted)

1. Solve algebraically: $\log_{15}(3-x)+\log_{15}(1-x)=1$. Justify the validity of each solution **(4 marks)**

2. Consider the graphs of $f(x)=\dfrac{x^2-x-6}{x^2-9}$ and

$$g(x)=\frac{x}{x^2-9}$$

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language. **(4 marks)**

Answers MULTIPLE-CHOICE

1. d	12. c	23. a	34. b
2. b	13. b	24. b	35. c
3. b	14. d	25. a	36. a
4. c	15. c	26. a	37. c
5. b	16. b	27. c	38. c
6. d	17. b	28. d	39. c
7. c	18. d	29. a	40. a
8. a	19. a	30. a	41. d
9. b	20. a	31. d	42. d
10. c	21. d	32. c	43. a
11. c	22. d	33. b	44. b

WRITTEN-RESPONSE (Calculator permitted)

1.

$$3\sin^2\theta + 5\cos\theta = 1$$

$$3(1-\cos^2\theta) + 5\cos\theta - 1 = 0 \qquad \leftarrow 1 \text{ mark}$$

$$3 - 3\cos^2\theta + 5\cos\theta - 1 = 0$$

$$-3\cos^2\theta + 5\cos\theta + 2 = 0$$

$$3\cos^2\theta - 5\cos\theta - 2 = 0$$

$$(3\cos\theta + 1)(\cos\theta - 2) = 0$$

$\frac{1}{2}$ mark $\longrightarrow$ $\cos\theta = -\frac{1}{3}$ ~~$\cos\theta = 2$~~ $\leftarrow \frac{1}{2}$ mark

$\frac{1}{2}$ mark $\longrightarrow$ $1.91 + 2\pi n, \quad n \in I$

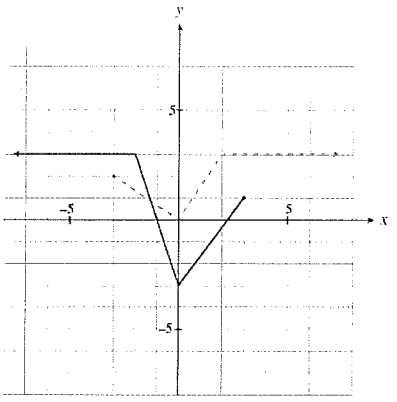
$\frac{1}{2}$ mark $\longrightarrow$ $4.37 + 2\pi n, \quad n \in I$

↑ 1mk

2.

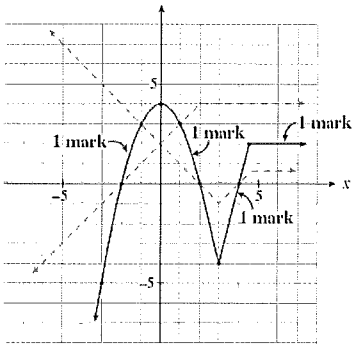
Left Side	Right Side
$\frac{\tan 2\theta(1-\tan\theta)\cos^2\theta}{\sin 2\theta}$	$\frac{1}{1+\tan\theta}$
1 mark → $= \frac{\frac{2\tan\theta}{(1-\tan^2\theta)}(1-\tan\theta)\cos^2\theta}{2\sin\theta\cos\theta}$	
1 mark → $= \frac{2\tan\theta(1-\tan\theta)\cos^2\theta}{(1-\tan\theta)(1+\tan\theta)2\sin\theta\cos\theta}$	
$\frac{1}{2}$ mark → $= \frac{2\sin\theta\cos^2\theta}{\cos\theta(1+\tan\theta)2\sin\theta\cos\theta}$	
$\frac{1}{2}$ mark → $= \frac{2\sin\theta\cos\theta}{1+\tan\theta 2\sin\theta\cos\theta}$	
1 mark → $= \frac{1}{1+\tan\theta}$	

3.



1 mark: vertical expansion
1 mark: reflection
1 mark: down 3
1 mark: ends/shape

4.



WRITTEN-RESPONSE (No calculators)

1. $\log_{15}(3-x)(1-x) = 1 \quad \leftarrow \frac{1}{2}$ mark

$$(3-x)(1-x) = 15 \quad \leftarrow \frac{1}{2} \text{ mark}$$

$$3 - 4x + x^2 = 15$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6, \quad x = -2 \quad \leftarrow \frac{1}{2} \text{ mark} + \frac{1}{2} \text{ mark}$$

$x = 6$ is rejected because either $3-x$ or $1-x$ will result in a negative number. The log of a negative number is not defined. ←1 mark

$x = -2$ is accepted because both $3-x$ and $1-x$ result in a positive number. The log of a positive number is defined. ←1 mark

2.

$f(x) = \frac{(x-3)(x+2)}{(x-3)(x+3)}$ $\qquad g(x) = \frac{x}{(x+3)(x-3)}, \quad x \neq \pm 3$

$= \frac{x+2}{x+3} \quad x \neq \pm 3$

	$f(x)$	$g(x)$	Similarity? (Y/N)
y-int:	$\frac{2}{3}$	0	N
x-int:	-2	0	N
domain:	$x \neq \pm 3$	$x \neq \pm 3$	Y
range:	$\mathbb{R}$	$\mathbb{R}$	Y
vertical asymptote:	$x = -3$	$x = 3, \ x = -3$	they share one asymptote with, $x=-3$
horizontal asymptote:	$y = 1$	$y = 0$	N
points of discontinuity	$(3, \frac{5}{6})$	none	N