Pre-Calculus 12 Practice Exam 2
MULTIPLE-CHOICE (Calculator permitted)

1. Solve $\cos 3x = \sin x,\ 0 \le x \le \frac{\pi}{2}$

   a. 0.39 b. 0.40, 1.25 c. 0.79 d. 0.79, 1.18

2. Determine the equation of a circle with centre $(0,0)$ passing through the point $P(-2,5)$

   a. $x^2 + y^2 = 3$ c. $x^2 + y^2 = 21$
   b. $x^2 + y^2 = 9$ d. $x^2 + y^2 = 29$

3. Determine the measure of the standard position angle θ if the point $P(-4,3)$ is on the terminal arm of angle θ, where $0° \le \theta < 360°$

   a. 37° b. 53° c. 127° d. 143°

4. Express as a single logarithm: $\log a - \log b - 3\log c$

   a. $\log\frac{a}{bc^3}$ b. $\log\frac{a}{b^3c^3}$ c. $\log\frac{ac^3}{b}$ d. $\log\frac{ac^3}{b^3}$

5. Determine the $x-$ intercept of $y = 5^x - 3$

   a. -2 b. 0.008 c. 0.6 d. 0.68

6. An investment earns 2.25% per annum compounded daily. How many years would be required for an investment to triple in value? Assume all years have 365 days.

   a. 4.88 b. 5.41 c. 48.83 d. 49.37

7. Determine the number of different arrangements of all the letters in the word TRIGONOMETRY.

   a. 4 989 600 c. 119 750 400
   b. 59 875 200 d. 479 001 600

8. An area code is the first 3 digits in a phone number and indicates the location of either the province or the city. In Canada, the following area codes exist:

Manitoba	204	Ontario	519, 613, 705, 807
Saskatchewan	306	Yukon and NW Territories	867
Québec (Québec City)	418	Toronto (Ontario)	289, 647
Montreal	514	Ontario (Toronto Metro)	416
Newfoundland	709	New Brunswick	506
Québec	450, 819	Alberta	780
British Columbia	250, 604, 778	Nova Scotia	902
Alberta (south)	403		

   Notice that there are 3 area codes for British Columbia: 250, 604 and 778. It will be necessary to add another area code as the population increases. The new area code cannot be the same as an existing code, it must begin with a 3 and end in an even number. Determine the number of possible area codes to choose from.

   a. 40 b. 44 c. 49 d. 50

9. In a standard deck of 52 cards, determine the number of 5-card hands that must contain at least 3 queens.

   a. 4512 b. 4560 c. 4704 d. 4752

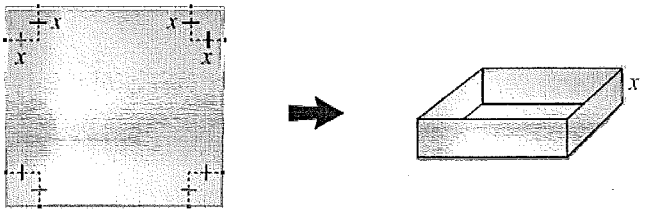
10. A dance group has twelve people from which five need to be chosen to compete in a national competition. Bob and Nancy are in the group of twelve and have recently obtained gold at a regional competition. They are therefore required to be among the five selected for the national competition. Given this requirement, how many different five-member teams are possible?

    a. 120 b. 220 c. 252 d. 792

11. Determine the 5th term in the expansion of $(3x+2y)^n$, where $n \ge 6$

    a. ${}_nC_4(3x)^{n-4}(2y)^4$ c. ${}_nC_4(3x)^4(2y)^{n-4}$
    b. ${}_nC_5(3x)^{n-5}(2y)^5$ d. ${}_nC_5(3x)^5(2y)^{n-5}$

12. A sheet of metal 12 cm × 12 cm will be used to make an open-top box by removing a square of length x in each corner and turning up the sides as shown in the diagram.



    What is the volume of the box as a function of x?

    a. $V = x^3$ c. $V = 144 - 4x^2$
    b. $V = x(12-x)^2$ d. $V = x(12-2x)^2$

13. Determine all solutions to the equation $\sqrt{x+4} = 3x$

    a. -0.61, 0.72 b. -0.61 c. 0.72 d. 1.33

14. Which equation represents the graph of $f(x)$ after it is horizontally stretched by a factor of $\frac{1}{2}$?

a. $y = f(2x)$ c. $y = f(\frac{1}{2}x)$

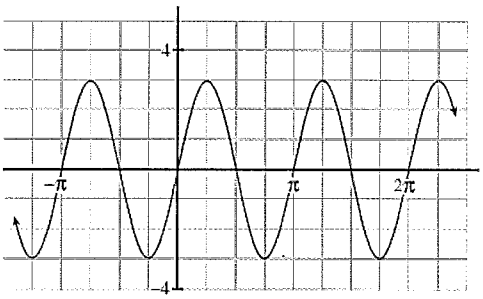
b. $y = 2f(x)$ d. $y = \frac{1}{2}f(x)$

MULTIPLE-CHOICE (Calculator NOT permitted)

15. Determine the equivalent expression for ${}_{22}C_8$

a. ${}_8C_{22}$ b. ${}_{20}C_6$ c. ${}_{22}C_{14}$ d. $\frac{{}_{22}P_8}{14}$

16. The graph of $y = a\sin bx$ is shown. Determine the values of a and b.



a. $a = -3,\ b = 2$
b. $a = -3,\ b = \pi$
c. $a = 3, b = 2$
d. $a = 3, b = \pi$

17. The terminal arm of angle θ in standard position intersects the circle at the point (x, y). Which expression represents $\cot\theta$?

a. $\frac{y}{x}$ b. $\frac{x}{y}$ c. x d. y

18. Which expression represents the measure of all angles in radians that are coterminal with angle θ?

a. $2\pi + n\theta, n \in Z$ c. $\theta + \pi n, n \in Z$

b. $\theta + \frac{\pi}{2}n, n \in Z$ d. $\theta + 2\pi n, n \in Z$

19. Determine the range of the function $y = -5\sin 2x - 3$

a. $-8 \le y \le 2$ c. $-5 \le y \le 5$

b. $-8 \le y \le -2$ d. $-2 \le y \le 8$

20. Determine the general solution for $\sin 4x = -1$

a. $x = \frac{\pi}{8} + \frac{\pi n}{2}, n \in Z$ c. $x = \frac{\pi}{8} + 2\pi n, n \in Z$

b. $x = \frac{3\pi}{8} + \frac{\pi n}{2}, n \in Z$ d. $x = \frac{3\pi}{8} + 2\pi n, n \in Z$

21. Chantal simplified the expression $\frac{\csc\theta + \sec\theta}{\sin\theta + \cos\theta}$ as shown below. In which step is Chantal's first error?

Steps	
1.	$\frac{\frac{1}{\sin\theta} + \frac{1}{\cos\theta}}{\sin\theta + \cos\theta}$
2.	$\frac{\frac{\cos\theta + \sin\theta}{\sin\theta}}{\sin\theta + \cos\theta}$
3.	$\left(\frac{\cos\theta + \sin\theta}{\sin\theta}\right)\left(\frac{1}{\sin\theta + \cos\theta}\right)$
4.	$\frac{1}{\sin\theta}$

a. 1
b. 2
c. 3
d. 4

22. Determine all non-permissible values for the expression $\frac{\sec x}{2\sin x + 1}$, in the interval $0 \le x < 2\pi$

a. $x = \frac{\pi}{2}, \frac{3\pi}{2}$ c. $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

b. $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ d. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

23. The function $h = -5\cos\frac{\pi}{60}t + 6$ gives Cassandra's height, h metres, above the ground when she is riding a Ferris wheel after t seconds. Determine the length of time for one rotation and the lowest point on the Ferris wheel.

a. 60 sec, 6 m c. 120 sec, 6 m

b. 60 sec, 1 m d. 120 sec, 1m

24. Determine the number of solutions for $\sin^2 x(\csc x + 1) = 0$ in the interval $0 \le x < 2\pi$ with the correct reasoning.

a. There are three solutions because $\sin^2 x = 0$ has two solutions and $\csc x + 1 = 0$ has one solution in the interval $0 \le x < 2\pi$

b. There are three solutions because $\sin^2 x = 0$ has one solution and $\csc x + 1 = 0$ has two solutions in the interval $0 \le x < 2\pi$

c. There is one solution because $\sin^2 x = 0$ has to be rejected and $\csc x + 1 = 0$ has one solution in the interval $0 \le x < 2\pi$

d. There is one solution because $\sin^2 x = 0$ has one solution and $\csc x + 1 = 0$ has to be rejected in the interval $0 \le x < 2\pi$

25. Two students, Yuri and Rubin, solved the exponential equation $2^{x+1}=3$ as shown below.

Yuri's Solution	Rubin's Solution
$2^{x+1}=3$ $\log 2^{x+1}=\log 3$ $(x+1)\log 2=\log 3$ $x\log 2+\log 2=\log 3$ $x\log 2=\log 3-\log 2$ $x=\frac{\log 3-\log 2}{\log 2}$	$2^{x+1}=3$ $x+1=\log_2 3$ $x=\log_2 3-1$

Which statement is true?

a. Yuri is incorrect, Rubin is incorrect.
b. Yuri is incorrect, Rubin is correct.
c. Yuri is correct, Rubin is incorrect.
d. Yuri is correct, Rubin is correct.

26. Which is the best estimation of $\log_3 30$?

a. 3.1 b. 3.4 c. 3.6 d. 3.9

27. Solve for x: $\log_2 3=2\log_8 x$

a. $3^{\frac{2}{3}}$ b. $3^{\frac{3}{2}}$ c. $2^{\frac{2}{3}}$ d. $2^{\frac{3}{2}}$

28. Determine the Richter scale reading for an earthquake that is 5 times more intense than another earthquake that measures 4.0 on the Richter scale.

a. 9 b. 20 c. $4+\log 5$ d. $5+\log 4$

29. Determine the domain of the function $y=\log(4-x^2)$

a. $-2<x<2$ c. $x<-2, x>2$
b. $-2\le x\le 2$ d. $x\le -2, x\ge 2$

30. Explain how the graph of $y-5=f(x)$ is related to the graph of $y=f(x)$

a. It is the graph of $y=f(x)$ translated 5 units up.
b. It is the graph of $y=f(x)$ translated 5 units down.
c. It is the graph of $y=f(x)$ translated 5 units to the left.
d. It is the graph of $y=f(x)$ translated 5 units to the right.

31. For which of the following does $f(x)=f(-x)$?

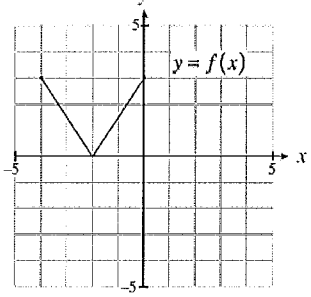
I.	$y=\sin x$
II.	$y=\cos x$
III.	$y=(x-3)^2$
IV.	$y=x^2+3$

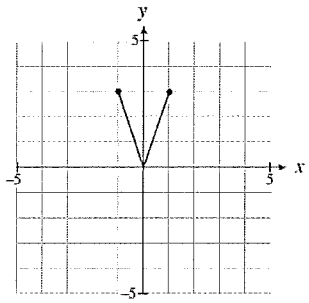
a. I, III only
b. I, IV only
c. II, III only
d. II, IV only

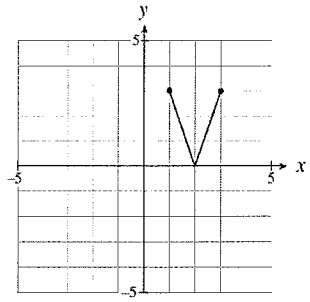
32. The point $P(-3,-8)$ is on the graph $y=f(x)$. Which point must be on the graph of $y=-f(x-5)$?

a. $(-8,-8)$ b. $(-8,8)$ c. $(2,8)$ d. $(8,-8)$

33. The graph of $y=f(x)$ is shown below. Which graph represents $y=f(2x-4)$?



a.


c.


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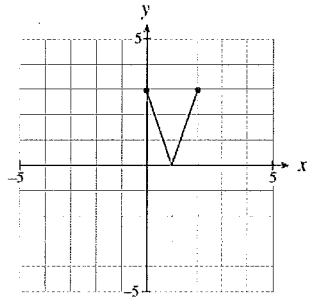
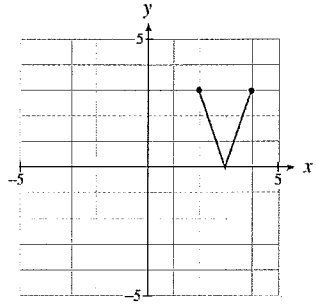


d.


34. A cubic polynomial function f has zeros {-3, 0 , 2}. Which restriction on the domain of f will allow its inverse to be a function?

a. $x>-3$ b. $x>0$ c. <0 d. $x>2$

35. Raj used synthetic division to divide a polynomial $f(x)$ by $x-2$ as shown below.

$$\begin{array}{r|cccc} 2 & 1 & -3 & k & -5 \\ \hline & & & & -1 \end{array}$$

Determine the value of k that will give a remainder of -1 as shown in the table.

a. 1 b. 4 c. 5 d. 6

36. Compare the graphs of the two functions at $x=2$ $f(x)=x(x-2)^3(x+2)$ and $g(x)=x(x-2)^2(x+2)$

a. The graph of $f(x)$ crosses the x-axis at $x=2$ and the graph of $g(x)$ just touches the x-axis at $x=2$ but does not cross it.
b. The graph of $f(x)$ just touches the x-axis at $x=2$ but does not cross it and the graph of $g(x)$ crosses the x-axis at $x=2$.
c. The graph of $f(x)$ crosses the x-axis at $x=2$ and the graph of $g(x)$ crosses the x-axis at $x=2$.
d. The graph of $f(x)$ just touches the x-axis at $x=2$ but does not cross it and the graph of $g(x)$ just touches the x-axis at $x=2$ but does not cross it.

37. Given the functions $f(x)=x+3$ and $g(x)=x^2-4$, determine the value of $(f+g)(-2)$.

a. 0 b. 1 c. 3 d. 5

38. For which of the following functions is $f(f(x))=x$, for all values of x in the domain of f?

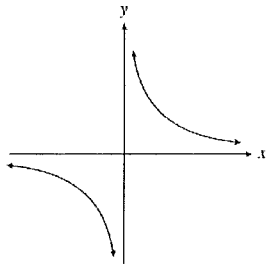
I.	$f(x)=x$
II.	$f(x)=-x$
III.	$f(x)=\frac{1}{x}$

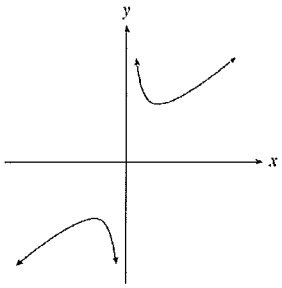
a. I and II only
b. I and III only
c. II and III only
d. I, II, and III

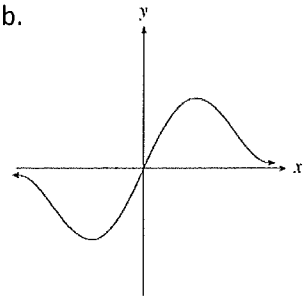
39. A polynomial function f has zeroes 1, -1, and 2. Given the function $g(x)=\frac{x+1}{x-2}$, determine the domain of the function $h(x)=\frac{f(x)}{g(x)}$.

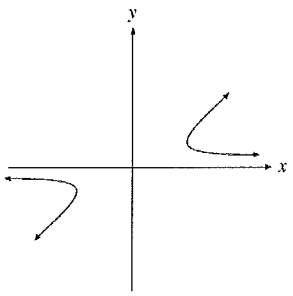
a. $x\in R$
b. $x\in R,\ x\neq 2$
c. $x\in R,\ x\neq -1$
d. $x\in R,\ x\neq -1,2$

40. Given $f(x)=x$ and $g(x)=\frac{1}{x}$, which graph best represents $y=f(x)+g(x)$?

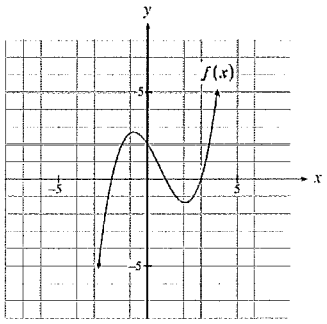
a.


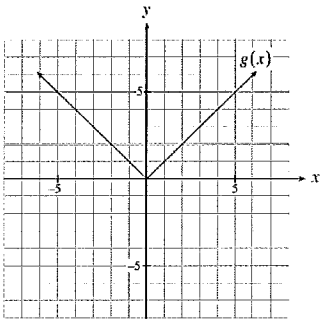
c.


b.


d.


41. The graphs of $f(x)$ and $g(x)$ are shown below. Determine $f(g(-3))$.





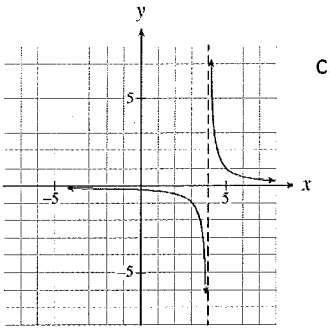
a. -6 b. -2 c. 0 d. 3

42. Determine the equations of all asymptotes of the graph of $y+2=\frac{1}{x-1}$

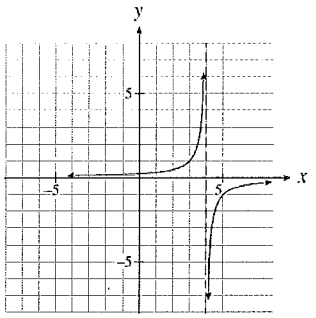
a. $x=-1, y=-2$
b. $x=-1, y=2$
c. $x=1, y=-2$
d. $x=1, y=2$

43. Which of the following best represents the graph of the rational function $y=\frac{x}{x^2-4x}$?

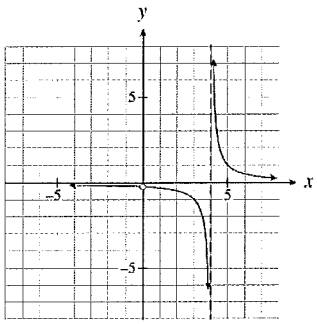
a.



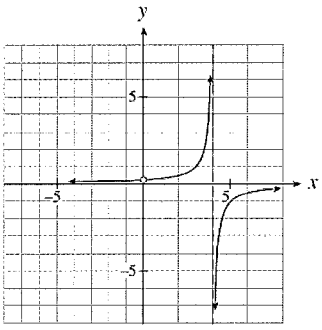
c.



b.



d.



44. For the function $f(x)=\dfrac{x^2-4}{x^2-2x}$, which of the following statements explain the behaviour of the graph of f for the values of a variable near a non-permissible value?

a. When x is close to 2 on either side, f is close to 2.
When x is just to the right of 0, f is a large positive value.
When x is just to the left of 0, f is a large negative value.

b. When x is close to 2 on either side, f is close to 4.
When x is just to the right of 0, f is a large positive value.
When x is just to the left of 0, f is a large negative value.

c. When x is close to 2 on either side, f is close to 2.
When x is just to the right of 0, f is a large negative value
When x is just to the left of 0, f is a large positive value.

d. When x is close to 2 on either side, f is close to 4.
When x is just to the right of 0, f is a large negative value
When x is just to the left of 0, f is a large positive value.

WRITTEN-RESPONSE QUESTIONS (Calculator permitted)

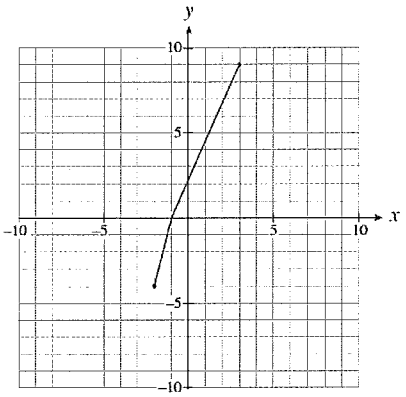
1. A food sample contains 300 bacteria. The doubling time for bacteria left at room temperature is 20 minutes. How many minutes will it take to reach an unsafe level of 100 000 bacteria?

   Solve algebraically using logarithms. Answer must be written as a decimal accurate to at least 2 decimal places. **(4 marks)**

2. Given $\sin\alpha=\dfrac{1}{5}$, where α is in quadrant I and $\cos\beta=\dfrac{2}{3}$, where β is in quadrant IV, determine the exact value of $\sin(\alpha-\beta)$. **(4 marks)**

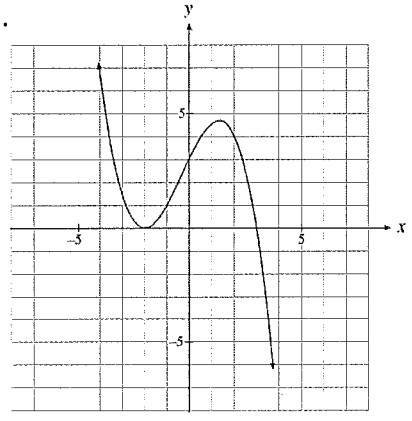
3. Prove algebraically:
   $\dfrac{\cos\theta}{1-\sin\theta}=\sec\theta+\sec\theta\csc\theta-\cot\theta$ **(4 marks)**

4. The graph of $y=f(x)$ is sketched below. Determine the domain and range of $y=\sqrt{f(x)}$ and explain how it was determined. **(4 marks)**



WRITTEN-RESPONSE QUESTIONS (Calculator NOT permitted)

1. Determine an equation for the cubic polynomial function graphed below. Leave answer in factored form. **(4 marks)**



2. Explain the relationship between the exponential function $f(x)=2^x+1$ and its inverse. Provide an answer that includes an algebraic analysis and describes graphical characteristics. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language. **(4 marks)**

Answers MULTIPLE-CHOICE

1. a	12. d	23. d	34. d
2. d	13. c	24. c	35. b
3. d	14. a	25. d	36. a
4. a	15. c	26. a	37. b
5. d	16. c	27. b	38. d
6. c	17. b	28. c	39. d
7. b	18. d	29. a	40. c
8. c	19. a	30. a	41. c
9. b	20. b	31. d	42. c
10. a	21. b	32. c	43. b
11. a	22. d	33. b	44. a

4.

For $y=\sqrt{f(x)}$ domain: $[-1, 3]$ ← **1 mark**

range: $[0, 3]$ ← **1 mark**

To determine the domain of $y=\sqrt{f(x)}$, only values of $f(x)$ that are positive can be considered which is where the range of $y=f(x)$ is positive. This occurs in the restricted domain $[-1, 3]$. **(1 mark)**

To determine the range of $y=\sqrt{f(x)}$, the lowest $f(x)$ value in the restricted domain is 0 and the largest value is 9. The square roots of these values are 0 and 3. That determines the range $[0, 3]$. **(1 mark)**

WRITTEN-RESPONSE (Calculator permitted)

1. ½ mark ½ mark

$100\,000 = 300(2)^{\frac{t}{20}}$ ← ½ mark

$\frac{1000}{3} = 2^{\frac{t}{20}}$ ← ½ mark

$\log\left(\frac{1000}{3}\right) = \frac{t}{20}\log 2$ ← 1 mark

$\frac{20\log\left(\frac{1000}{3}\right)}{\log 2} = t$ ← ½ mark

$167.62 = t$ minutes ← ½ mark

2. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{1}{5}\right)\left(\frac{2}{3}\right) - \left(\frac{2\sqrt{6}}{5}\right)\left(\frac{-\sqrt{5}}{3}\right)$

½ mk ½ mk 1 mk 1 mk

$= \frac{2}{15} + \frac{2\sqrt{30}}{15}$

$= \frac{2+2\sqrt{30}}{15}$

← 1 mark

3.

Left Side	Right Side
$\frac{\cos\theta}{1-\sin\theta}$	$\sec\theta + \sec\theta\csc\theta - \cot\theta$
	$= \frac{1}{\cos\theta} + \frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$ ← 1 mark
	$= \frac{\sin\theta + 1 - \cos^2\theta}{\sin\theta\cos\theta}$ ← ½ mark
	$= \frac{\sin\theta + \sin^2\theta}{\sin\theta\cos\theta}$ ← ½ mark
	$= \frac{\cancel{\sin\theta}(1+\sin\theta)}{\cancel{\sin\theta}\cos\theta}$ ← ½ mark
	$= \frac{1+\sin\theta}{\cos\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta}$ ← ½ mark
	$= \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}$ ← ½ mark
	$= \frac{\cos^2\theta}{\cancel{\cos\theta}(1-\sin\theta)}$
	$= \frac{\cos\theta}{1-\sin\theta}$ ← ½ mark

WRITTEN-RESPONSE (No calculators)

1. $y = a(x+2)^2(x-3)$ ← **2 marks**

passing through $(0, 3)$

$3 = a(0+2)^2(0-3)$ ← **1 mark**

$3 = a(-12)$

$-\frac{1}{4} = a$ ← ½ **mark**

$y = -\frac{1}{4}(x+2)^2(x-3)$ ← ½ **mark**

2.