Final Exam Review – Pre-Calculus 12

Chapter 1 – Transformations

1.1 Translation Functions Left\Right and Up\Down

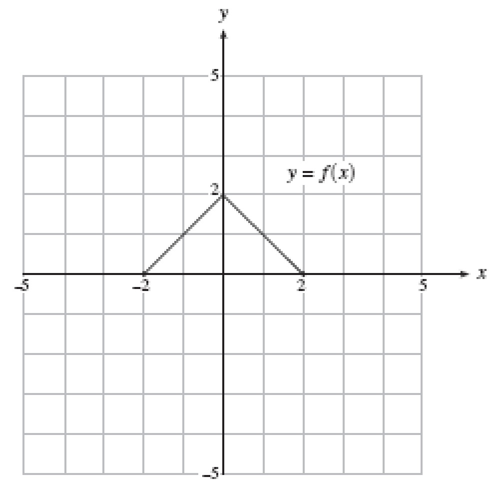
a. If the point (2, 3) is on $y = f(x)$ then what point must be on:

$y = f(x-2)$	$y = f(x+4)$	$y = f(x)-3$	$y = f(x)+5$
$y+4 = f(x)$	$y-9 = f(x)$	$y = f(x-9)+3$	$y-8 = f(x+5)$

b. If (4, 5) is on $y = f(x)$ and (7, 10) is on the translated graph find values of *a* and *b* that satisfy:

$y = f(x+a)+b$	$y+b = f(x+a)$

c. Given the graph $y = f(x)$ shown below. Graph $y = f(x+2)-3$



1.2 Reflections:

a. Match change in the function on the left with the change to the graph on the right.

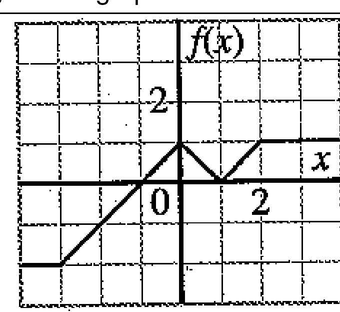
$y = f(x) \rightarrow y = f(-x)$	Reflect over line *y=x*
$y = f(x) \rightarrow y = -f(x)$	Reflect over y-axis
$y = f(x) \rightarrow y = f^{-1}(x)$	Reflect over x-axis

b. If the point (4, -5) is on the graph $y = f(x)$, then what point must be on:

a. $y = f(-x)$	b. $-y = f(x)$ can this be written another way?
c. $y = -f(-x)$	c. $x = f(y)$ can this be written another way?
d. $-x = f(y)$	e. $-y = f(x+3)$

c. If the point (5, 4) is on the curve $y = f(x)$, then what values of *a* and *b* will move the point to (-5, -4) on the curve $y = af(bx)$?

d. Given the graph $y = f(x)$ shown on the right, sketch graphs of:

$y = f(-x)$ $-y = f(x)$ $y = f^{-1}(x)$	

1.3 Horizontal\Vertical Expansions & Compressions

a. Match the statement on the left with the equations on the right.

Horizontal Expansion by 2	$y = f(x) \rightarrow y = 2f(x)$

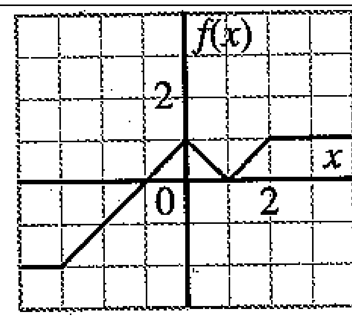
Horizontal Compression by 1/2	$y=f(x) \rightarrow y=\frac{1}{2}f(x)$
Vertical Expansion by 2	$y=f(x) \rightarrow y=f(2x)$
Vertical Compression by 1/2	$y=f(x) \rightarrow y=f\left(\frac{1}{2}x\right)$ or $y=f\left(\frac{x}{2}\right)$
	$y=f(x) \rightarrow \frac{1}{2}y=f(x)$ or $\frac{y}{2}=f(x)$
	$y=f(x) \rightarrow 2y=f(x)$ or

b. If the point (2, 5) is on the curve $y=f(x)$, then what point must be on:

$y=f(4x)$	$y=3f(x)$	$y=\frac{1}{2}f(x)$	$y=f\left(\frac{1}{5}x\right)$	$y=2f(6x)$

c. If the point (10, 12) is on the curve $y=af(bx)$ and the point (5, 24) is on the curve $y=f(x)$, then what are the values of *a* and *b*?

d. Given the graph $y=f(x)$ shown on the right, sketch graphs of:

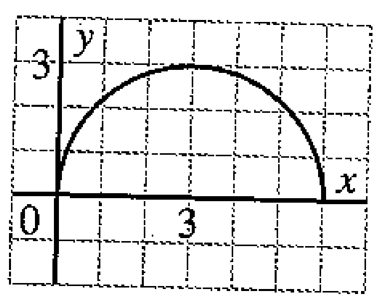
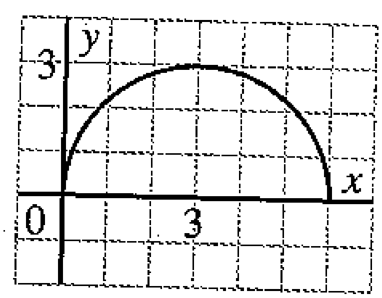
$y=f(2x)$ $y=\frac{1}{2}f(x)$ $y=4f\left(\frac{1}{2}x\right)$	

1.4 Combining Transformations

a. If the point (4, 6) is on the curve $y=f(x)$, then what point must be on:

$y=f(2(x-3))$	$y=f(2x+4)$	$y=-f(3x+9)-5$
$y=3f(2x-4)-1$	$2y=f(5x-10)+8$	$y=-3f\left(-2x+6\right)-4$

b. Given the graph of $y=f(x)$, shown below. Graph:

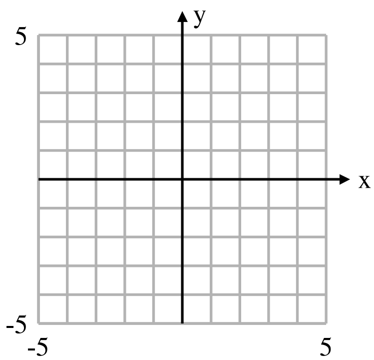
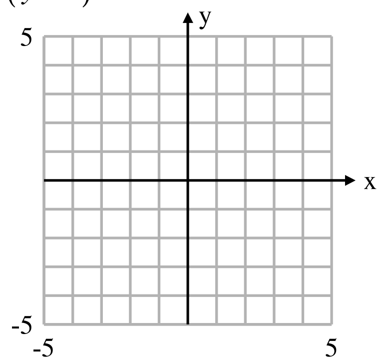
$y=f(2(x+3))$ 	$y=f(2x+3)$ 

1.5 More on inverses

a. Find the algebraic inverse:

$y=\frac{2}{3}x-4$	$y=5(x-3)^3$	$y=\frac{x-2}{5x-3}$

b. Complete the table:

Graph $y=(x+3)^2-2$ 	Graph $x=(y+3)^2-2$ 
Domain:	Domain:
Range:	Range:
Is this a function	Is this a function

Chapter 2 Fun with Functions

2.1 The Remainder Theorem

If $x-a$ divides a polynomial, $p(x)$, then the remainder is $p(a)$

a. Find the remainder to:

$\left(x^5+3x^2-1\right)\div(x+2)$	$\left(x^3-x+1\right)\div(x+1)$	$\left(x^{99}+1\right)\div(x-1)$

b. Find the point (x, y) on the graph of $y=p(x)$, if:

$p(x)=x^5+3x^2-1$ and $x=-2$	$p(x)=x^3-x+1$ and $x=-1$	$p(x)=x^{99}+1$ and $x=1$

c. Use long/synthetic division to confirm your answers to a.

$\left(x^5+3x^2-1\right)\div(x+2)$	$\left(x^3-x+1\right)\div(x+1)$

2.2 The Factor Theorem

a. Create a list of the possible factors of:

$x^3-3x^2-4x+12$	x^3-5x^2+8x-4	$x^4-6x^3+13x^2-12x+4$

b. Factor:

$x^3-3x^2-4x+12$	x^3-5x^2+8x-4	$x^4-6x^3+13x^2-12x+4$

2.3 Graphing Polynomials

a. Identify the following functions as polynomials (P) or not polynomials (NP) (and say why it is a NP)

$y=x^2+3x+6$	$y=x^{3.2}+6x+9$	$y=\frac{1}{\sqrt{2}}x^3-5x+9$

b. Graph

$y=x^3-3x^2-4x+12$	$y=x^3-5x^2+8x-4$	$y=x^4-6x^3+13x^2-12x+4$

c. Sketch a rough graph (degree, co-efficient of leading term, *y*-intercept)

$y=1.3x^6+5x^2-8$	$y=-2x^3+4x^2-8x+9$	$y=2.6x^5-4x^3+2x^2+3x+2$

d. A graph of a polynomial function has *x*-intercepts at 3, 2, and -1. It has a *y*-intercept of -24. Create the polynomial function

e. Graph:

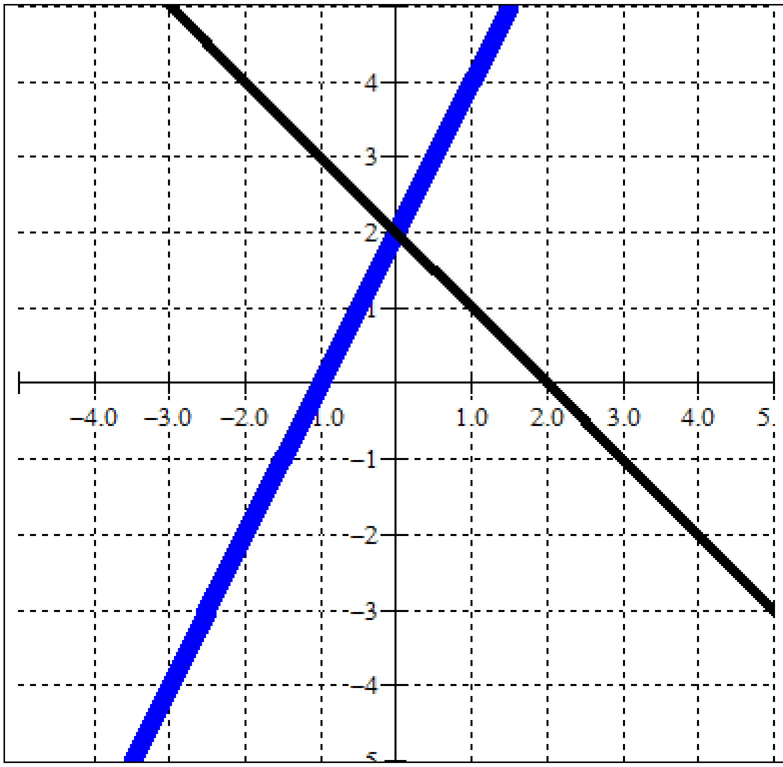
$y=\left(x^3-3x^2-4x+12\right)(x+2)$	$y=\left(x^4-6x^3+13x^2-12x+4\right)(x-3)$

<u>2.4 Operations with Functions</u>

a. Given the function $f(x)=x-1$, and $g(x)=2x+3$, determine:

$f(x)+g(x)$	$g(x)-f(x)$	$f(x)\times g(x)$	$\dfrac{f(x)\times f(x)}{g(x)}$

b. Given the graphs of $y=f(x)$ (thick line) and $y=g(x)$ (thin line), sketch graphs of:



$y=f(x)+g(x)$

$y=f(x)\times g(x)$

$y=\dfrac{f(x)}{g(x)}$

<u>2.5 Composition of Functions</u>

a. If $f(x)=x^2-9$ and $g(x)=x+1$, find:

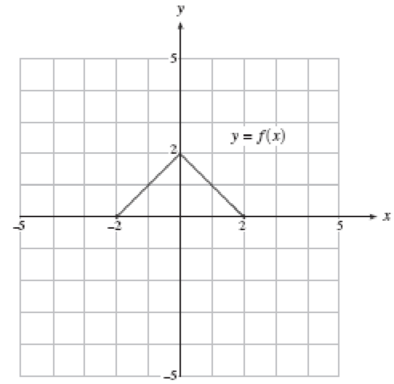
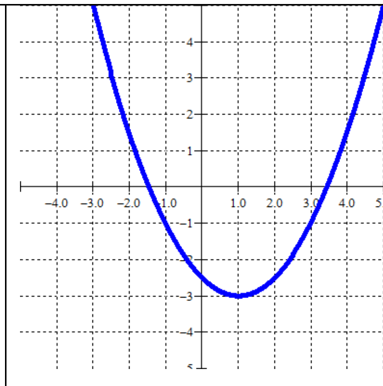
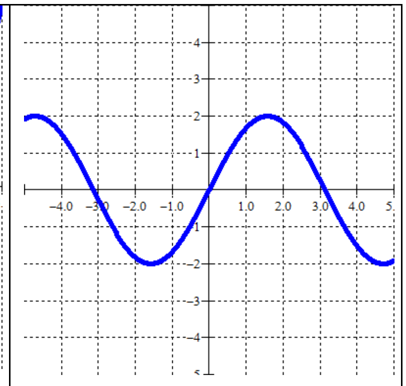
$f\left(g(2)\right)$	$g\left(f(4)\right)$	$f\left(g(x)\right)$	$g\left(f(x)\right)$

b. The surface area of a cube is: $S=6s^2$ and it's volume is: $V=s^3$, change the volume formula of the cube so that the volume is a function of it's surface area

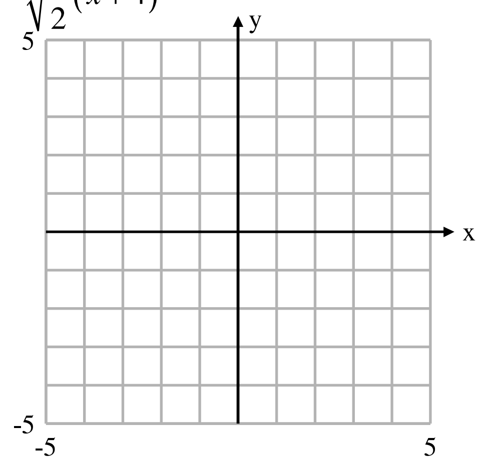
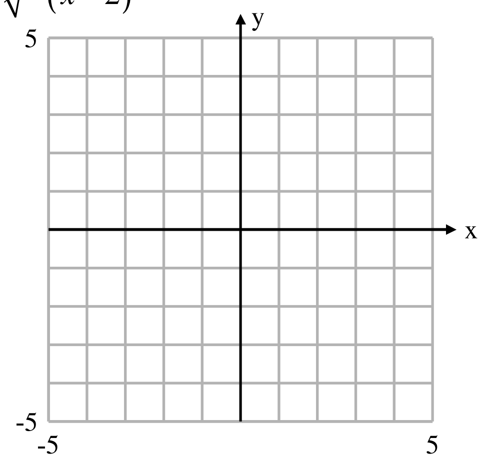
c. The diagonal of a square $d=\sqrt{2}s$ and the area of a square is $A=s^2$, change the diagonal formula of a square so that the diagonal formula is a function of it's area

2.6 Radical Functions

a. Given the graph of $y = f(x)$, shown below, sketch a graph of $y = \sqrt{f(x)}$, and state the domain and range of $y = \sqrt{f(x)}$.

		
Domain	Domain	Domain
Range	Range	Range

b. Graph the following functions and state the domain & range

$y = -\sqrt{\frac{1}{2}(x+4)}$ 	$y = \sqrt{-(x-2)}$ 
Domain	Domain
Range	Range

2.7 Rational Functions

a. Find the properties of the following functions and then graph them

Properties: 1. Locations of Vertical Asymptotes (x = #) 2. Location of Horizontal Asymptote (y = #) 3. Location of Oblique/Slant Asymptote (y = mx + b) 4. Location of the hole (x, y) 5. Location of y-intercept (0, y) 6. Location of x-intercept(s) (x, 0)	$y=\frac{x^2+3x+2}{2x^2+7x+5}$ $y=\frac{x^2-3x-4}{x-3}$ $y=\frac{(x-1)(x+2)(x-3)}{(x-1)(x-3)}$ $y=\frac{(x-1)(x+2)(x-3)}{(x+3)(x-2)}$

Chapter 3 – Logarithms

3.1 Using Exponential Functions

Won't really be assessed. You need to know these formulas to solve problems found later in the review

$A=P\left(1+\frac{i}{n}\right)^{t\times n}$	$A=P(X)^{\frac{t}{n}}$

3.2 Definition of a Logarithm

a. Convert between exponential and logarithmic form

Write in exponential form: $\log_2 8=3$	Write in logarithmic form: $5^3=125$
Write in exponential form: $\log_p q=r$	Write in logarithmic form: $x^y=z$

b. Evaluate logarithms with out a calculator:

$\log_3 27$	$\log_2 \frac{1}{16}$	$\log_{81} 3$	$\log_8\left(\frac{1}{2}\right)$

c. Estimate logarithms with out a calculator:

$\log_2 7$	$\log_3 83$	$\log_5 22$	$\log_{16} 3$

3.3 Laws of Logarithms (no calculator anywhere folks)

a. Evaluate: (Using the change of base rule)

$4\log_{5.6} 84$ (use a calculator for this one ☺)	$\log_{\sqrt{8}} 16$

b. Write as a sum/difference of logarithms (split those darn things up!)

$\log\left(\frac{1000a^3b^3}{c^2\sqrt[4]{d^5}}\right)$	$\log_2\left(\frac{x^2y^3}{8z^4}\right)$

c. Write as a single logarithm (squish them all together!)

$2\log a+4\log b-5\log c-\frac{1}{2}\log d$	$2+\log_3 a+2\log_3 b-\log_3 c$

d. If $\log 5=x$ and $\log 4=y$, write in terms of *x* and *y*

$\log 20$	$\log\left(\frac{5}{4}\right)$	$\log 10$	$\log_5 4$

3.4 Solving problems that involve logarithms:

a. How long does it take for \$250 take to grow to \$570 if it is invested in a saving account paying 4.75% p.a. compounded monthly?

b. A radioactive sample of Carbon 16 has a half life of 50 days. How long does it take for another sample of Carbon 16 to decay from 70 grams to 12 grams?

c. A population of bunnies grows from 1200 to 5700 over a period of 70 days. Find the tripling time of this population.

d. The sound at a rock concert decreases by 10% for every 5 meters traveled. How far does it take for a volume of 2000 decibels to decrease to 20 decibels?

3.5 Earthquakes, pH & Growth.

a. Earthquake A – Magnitude 10.7. Earthquake B – Magnitude 6.8. How many times stronger is earthquake A compared to earthquake B?

b. Earthquake A – Magnitude 6.5. Find the magnitude of Earthquake B if:

Earthquake B is 2.5 times as strong (intense)

Earthquake B is $\frac{1}{3}$ times a strong (intense)

c. Solution A has a pH of 5.6. Solution B has a pH of 8.2. How many times more alkaline is solution B compared to solution A?

d. Solution A has a pH of 8.2. Solution B has a pH of 7.5. How many times more acidic is solution B compared to solution A?

e. Solution A has a pH of 7.9. Find the pH of solution B if:

Solution B is 5 times as acidic

Solution B is 3 times as alkaline

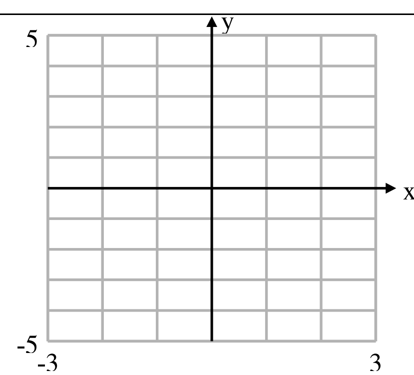
Solution B is half as acidic

f. A population of cats triples every 40 weeks. By how much does the population grow between the 20th and 90th week?

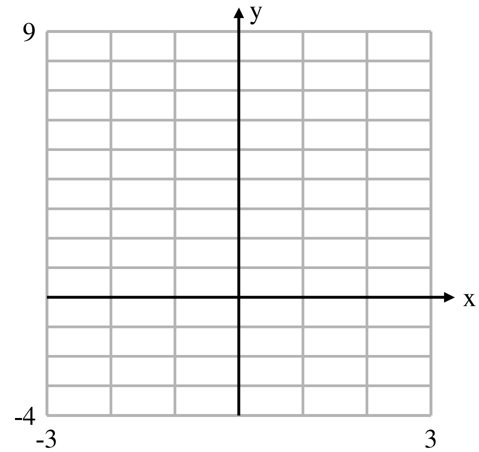
g. A population of bears grows by 20% every 15 days. By how much does the population grow between the 3rd and 50th day?

3.6 Graphing Exponential Functions (No calculators here folks!)

a. Given the function: $y = 2^x - 3$,

Range?	Asymptote	Graph
		

b. Given the function: $y = \left(\frac{1}{2}\right)^x + 1$

Range?	Asymptote	Graph
		

c. Find the inverse algebraically:

$y = 3^{x-4} + 8$	$y = 5^{2x-7} - 3$

<u>3.7 Solving Exponential Equations:</u>

a. Solve:

$8^{x+4} = 16^{2x}$	$5^{x-1} = 7^{x+3}$	$a^{x+3} = bc^{2x}$

<u>3.8 Graphing Logarithmic Functions</u> (No Calculators here!)

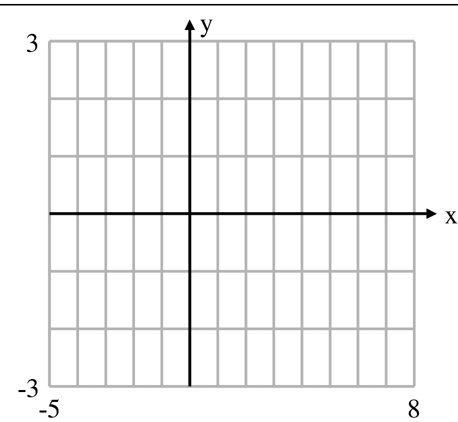
a. Find the domain without graphing:

$y = \log_6\left(-(x+4)\right)$	$y = \log_{x-2}\left(9 - x\right)$

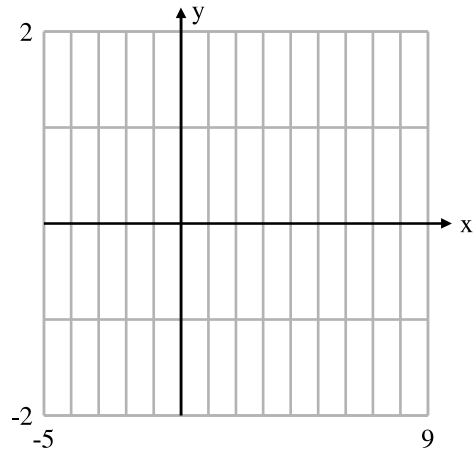
b. Find the inverse:

$y = \log_7\left(x-4\right) + 2$	$y = \log_3(x-4) - 9$

c. Given the function: $y = \log_2\left(x-2\right)$,

Domain?	Asymptote	Graph
		

d. Given the function: $y = \log_{\frac{1}{3}}(x+2)$,

Domain?	Asymptote	Graph
		

<u>3.9 Solving Logarithmic Equations</u>

a. Simplify:

$81^{\log_3 2x}$	$a^{\log_a 5x^2 + \log_a 2x}$	$\log_5 5^{2x} + \log_5 5^{3x} - \log_5 5^x$

b. Solve:

$\log_2 8 + \log_3 9 = \log x$	$\log_4(\log_3 x) = \frac{1}{2}$	$36^{\log_2 x} = 216$
$\log_3(x+4) + \log_3(6-x) = 2$	$2\log(3-x) = \log 4 + \log(6-x)$	$(\log_2 5)(\log_5 x) = 3$

Chapter 4 – Trigonometry of the Unit Circle

<u>4.1 – Radian Measure</u>

a. Convert Radians into Degrees:

3π	$\frac{5}{9}\pi$	1.25 radians

b. Convert Degrees into Radians

$130°$ (express as a fraction of π)	$260°$ (express as a fraction of π)	$283°$ (don’t express as a fraction of π, round correctly to 2 decimal places)

c. Use the arc length formula (ArcLength = Radius x Angle (in Radians))

An angle of $\frac{2}{3}\pi$ Radians subtends an arc of 20 cm. What is the radius of this circle?
A circle with a radius of 10 cm has a central angle of $130°$, what is the length of the arc it subtends?
A circle with radius of 25 cm subtends an arc of 50 cm. What is the measure ofthis angle in radians and in degrees?

<u>4.2 – Angles in Standard Position Part I</u>

a. Sketch the angle, find 2 coterminal angles and the reference angle for:

$130°$	$-170°$	$\frac{7}{9}\pi$	$-\frac{4}{5}\pi$	$\frac{17}{6}\pi$

4.3 – Angles in Standard Position in Part II

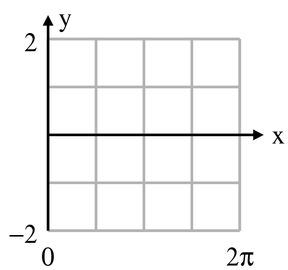
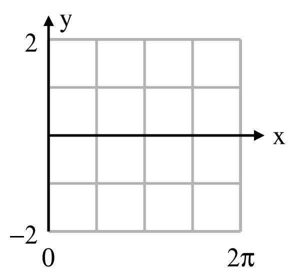
$\sin\theta = \frac{y}{r}$	$\cos\theta = \frac{x}{r}$	$\tan\theta = \frac{y}{x}$
$\csc\theta = \frac{r}{y}$	$\sec\theta = \frac{r}{x}$	$\cot\theta = \frac{x}{y}$

a. If $\sin\theta = -\frac{5}{12}$ and $\pi < \theta < \frac{3}{2}\pi$, find the exact value of $\tan\theta$

b. If $\cot\theta = -\frac{2}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the exact value of $\sec\theta$

c. What quadrant are you in if: $\sin\theta < 0$ and $\sec\theta < 0$?

4.4 – Special Triangles (You'd better memorize them soon!) and the graphs of sine and cosine
Draw the two special triangles and the graphs of sine & cosine

a. Using the graphs and special triangles, determine **the exact value of**:

$\sin\left(\frac{\pi}{3}\right)$	$\cos\left(\frac{11\pi}{6}\right)$	$\tan\left(-\frac{5}{4}\pi\right)$	$\sec\left(\frac{7\pi}{3}\right)$
$\sin\left(\frac{3}{2}\pi\right)$	$\sec\left(\frac{\pi}{2}\right)$	$\cot\left(\frac{\pi}{4}\right)$	$\sin\left(-\frac{45}{6}\pi\right)$

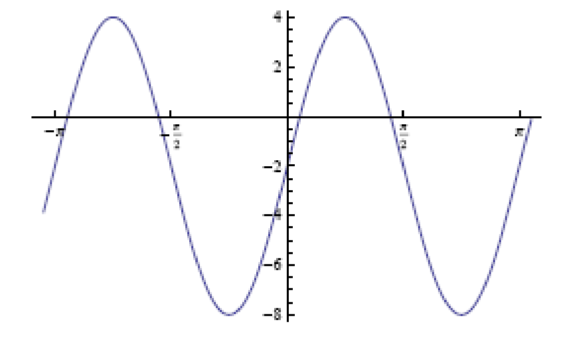
Chapter 5 – Graphing Trigonometric Functions

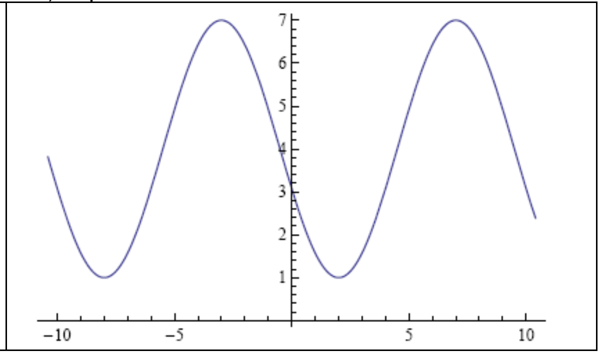
5.2, 5.3, 5.4 Graphing Sine & Cosine

a. List all of the characteristics of the following functions:
(Amplitude, Vertical Displacement, Period, Phase Shift, Maximum, Minimum)

$y = 6\sin\left(3\left(x - \frac{\pi}{3}\right)\right) - 2$	$y = -3\cos(5x - 20\pi) + 8$
$y = 2\sin\frac{2\pi}{14}x + 8$	$y = -4\cos\frac{\pi}{9}(x-2) + 4$

b. Given the graph, write the sinusoidal (sine/cosine) equation:





c. A sinusoidal function has a maximum at (2, 10) and its next minimum (5, -2). Find an equation that represents this situation

d. A sinusoidal function has a zero at (5, 0) and its next minimum is (7, -4). Find an equation that represents this situation.

e. A sinusoidal function has a maximum at $\left(\frac{\pi}{4}, 10\right)$ and its next minimum is $\left(\frac{7}{8}\pi, -2\right)$. Find an equation that represents this situation.

<u>5.5 Modeling with sine/cosine.</u>

a. A Ferris wheel has a diameter of 30 m. The bottom of the wheel is 1.5 m off the ground. It takes 3.5 minutes to do one complete revolution. A person gets on the Ferris wheel at its lowest point at time $t = 0$.

Write an equation that represents a person's height above the ground (h) at any time (t).
How high off the ground is the person at $t = 25$ seconds?
How long (in one rotation) is the person above 27 m?

b. In Vancouver on a certain day, high tide is 20 m at 2AM. The next low tide is 8 m at 6 AM.

Write an equation that represents the height (h) of the water at any time (t) (since midnight)
What is the height of the water at 8:45 AM?
What is the height of the water at 8:30 PM?

Chapter 6 – Trigonometric Proofs & Equations

<u>6.1 & 6.2 Reciprocal and Pythagorean Identities</u>

a. Evaluate:

$\csc\left(\frac{11}{7}\pi\right)$	$3\sec(11.45)$

b. Simplify:

$\sin x \csc x + \sec x \cos x$	$\cos^2 x \csc^2 x + 1$	$\frac{\cos x}{1-\sin x} + \frac{\cos x}{1+\sin x}$

c. Prove:

$\frac{1+\csc x}{\cot x + \cos x} = \frac{\cot x}{\csc x - \sin x}$	$\frac{\tan\theta}{\sec\theta + 1} = \frac{\sec\theta - 1}{\tan\theta}$

<u>6.3 Sum and Difference Identities</u>

a. Simplify (write as a single trig. statement)

$\sin 5x \cos 2x - \cos 5x \sin 2x$	$\cos 3x \cos 4x + \sin 3x \cos 4x$

b. If $\sin\theta = -\frac{5}{13}$ and $\frac{3}{2}\pi < \theta < 2\pi$, find the exact value of:

$\sin\left(\frac{\pi}{2}+\theta\right)$	$\cos\left(\theta+\frac{\pi}{6}\right)$	$\tan\left(\theta+\frac{\pi}{4}\right)$

6.4 Double Angle Identities

a. Simplify (write as a single trig statement)

$10\sin 3x\cos 3x$	$10\cos^2 8x - 5$	$4\cos^2 7x - \sin^2 7x$	$\frac{2\tan(5x)}{1-\tan^2(5x)}$

b. If $\cos\theta = -\frac{5}{13}$ and $\pi < \theta < \frac{3}{2}\pi$, find the exact value of:

$\sin 2\theta$	$\cos 2\theta$	$\tan(2\theta)$

c. Prove:

$\frac{\cos 2x}{\sin x} = \frac{\cot^2 x - 1}{\csc x}$	$\frac{\sin 2x}{1+\cos 2x} = \frac{\sec^2 x - 1}{\tan x}$

6.5 Restrictions

a. Find the restrictions in terms of $\sin x$ and $\cos x$

$\frac{\tan x}{\csc x - 1}$	$\frac{\cos x}{3\csc x - 4}$	$\frac{\cot x}{3\sin x - 2}$

6.6 Solving 'simple' Trigonometric Equations:

a. Solve the following equations (as exact values where possible)

$2\sin x - \sqrt{3} = 0$, $0 < x \le 2\pi$	$2\cos x + 1 = 0$, $-\pi \le x < \pi$	$5\cot x - 1 = 0$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

6.7 Solving trigonometric equations by factoring:

a. Solve (by factoring a common term):

$2\sin x\cos x + \cos x = 0$, $0 < x \le 2\pi$	$\sqrt{3}\tan x\sin x + \sin x = 0$, $-\pi \le x < \pi$

b. Write the general solution for questions in a.

c. Solve (by trinomial factoring):

$6\sin^2 - \sin x - 2 = 0$, $-\pi \le x < \pi$	$\sec^2 x + \sec x - 6 = 0$, $0 < x \le 2\pi$

d. Solve over the real numbers the question in c.

6.8 Double angle equations:

a. Solve, write answers as a general solution

$4\sin 5x\cos 5x = 1$	$2\cos^2 6x - 2\sin^2 6x = \sqrt{3}$

6.9 Solving Using the graphing calculator

a. Use the graphing calculator to solve:

$3\tan x = 2^x$, $-\pi \le x < \pi$	$3\cos x = \log x$, $0 < x \le 2\pi$

Chapter 7 – Permutations & Combinations

7.1 The Fundamental Counting Principle

a. Kasey goes to Wendy's to order a value meal. She has a choice of toppings, side, and drink

Toppings: Lettuce, Pickle, Tomato

Side: Fries, Baked Potato, Chili
Drink: Pop, Iced Tea
If she only has 1 topping, side, & drink how many value meals can she order?
b. How many ways can you order the letters of the word: HORSE?
c. A 10 question multiple choice test has 5 choices for each question (A, B, C, D, E), how many different ways can a person answer this test?

7.2 Permutations (Given on Formula sheet: ${}_nP_r = \dfrac{n!}{(n-r)!}$

a. There are 20 horses in a race. If you are buying tickets to predict the first two horses in a race, how many tickets must you buy?
b. There are 30 desks in a classroom, how many different seating plans are there for 25 students?
c. Solve

${}_nP_2 = 210$	${}_{17}P_r = 57120$

d. Simplify:

$\dfrac{(n-2)!}{(n+1)!}$	$\dfrac{(n+1)!(n-1)!}{(n!)^2}$	$\dfrac{(2n)!}{(2n-2)!}$

7.3 Permutations of Like Objects (Formula not given: $\dfrac{n!}{a!b!c!...}$)

a. At a math 12 study session there are 25 students. Mr. Epp brings 1 slice of pizza for each student. 10 slices are pepperoni, 12 are vegetarian, and the rest are Hawaiian. How many different ways can he give out the pizza?
b. How many ways are there to arrange the letters in the word: RACECAR?
c. Evaluate (with out a calculator): $\dfrac{7!}{3!5!}$

7.4 Combinations(Formula given: ${}_nC_r = \dfrac{n!}{(n-r)!r!}$)

The card game Euchre uses only a portion of a standard deck of cards. To play this game you need 9, 10, Jack, Queen, King, & Ace of all 4 suits
a. If 4 cards are dealt, how many ways are there to deal 3 red cards and 1 black card?
b. If 4 cards are dealt, how many ways are there to deal at least 3 red cards?
c. If 4 cards are dealt, how many ways are there to deal at most 1 black card?

d. Solve algebraically:

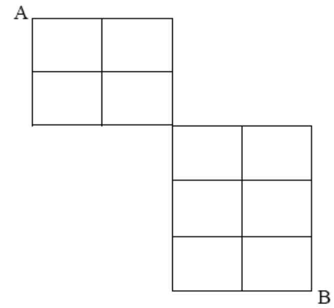
${}_nC_2 = 66$	${}_7C_n = 35$	$\dfrac{n!}{(n-2)!3!} = 7$

7.5 Pascal's Triangle:
a. How do you use combinations to find the element in the 320th row, 52nd diagonal of Pascal's Triangle?

b. Fill in the blanks:

${}_nC_0 = \_\_$	${}_{\_}C_8 = 1$	${}_{10}C_3 = {}_{\_}C_{\_}$	${}_{\_}C_1 = 25$	${}_{50}C_{24} = {}_{\_}C_{\_} + {}_{\_}C_{\_}$

c. How many ways are there from A to B in the map below (right & down only)



7.6 Binomial Theorem (Formula given: $t_{k+1} = {}_{n}C_{k}a^{n-k}b^{k}$)

a. Given the binomial: $\left(x-2y\right)^{12}$, determine the:

Number of terms	The coefficient of the term that has: $x^{3}y^{9}$
First 3 terms:	

b. The 4th term of the binomial expansion of $\left(3x-5\right)^{n}$ is $BLARGx^{12}$. Find the value of *n*.