

Operations with Functions

$$\textcircled{1} f(x) + g(x) = (3x-1)^2 + (x+2)^2 = 10x^2 - 2x + 5$$

$$f(x) - g(x) = 8x^2 - 10x - 3$$

$$f(x) \div g(x) = \frac{(3x-1)^2}{(x+2)^2} \quad , x \neq -2$$

$$\textcircled{2} g(f(3)) = g(11) = 2 \cdot 11 - 5 = 22 - 5 = 17$$

$$\textcircled{3} f(g(x)) = (3x-1)^2 - 4(3x-1) = 9x^2 - 18x + 5$$

$$\textcircled{4} g\left(g\left(-\frac{2}{3}\right)\right) = g\left(2 + \frac{10}{3}\right) = g\left(\frac{16}{3}\right) = 2 - 5 \cdot \frac{16}{3} = \frac{6-80}{3} = -\frac{74}{3}$$

$$\textcircled{5} f(g(x)) = \sqrt{6(2x+3)} = \sqrt{12x+18}$$

$$\textcircled{6} f(g(x)) = \sqrt{x+1} - 2 \quad ; x \geq -1$$

$$\textcircled{7} A(r) = \pi r^2 \quad A(c) = \pi \left(\frac{c}{2\pi}\right)^2 = \pi \cdot \frac{c^2}{4\pi^2} = \frac{c^2}{4\pi}$$
$$r(c) = \frac{c}{2\pi}$$

CH. 1.4-1.6 Transformations of Functions

$$\textcircled{1} y \mapsto -y \quad \textcircled{b}$$

$$\textcircled{2} \textcircled{b}$$

$$\textcircled{3} (a, b) \rightarrow \boxed{2}$$

\swarrow
 $\times \frac{1}{3}$

$$\textcircled{4} y = f(-(x-1)) + 3$$

$$(-2, 5) \rightarrow (2, -5) \rightarrow (3, -5) \rightarrow (3, -2)$$

$$\textcircled{5} \text{a) } y = \frac{1}{2} f[-(x-1)] - 2 \quad \text{b) } y = -f[2(x-1)] + 1$$

$$(6) \quad y \mapsto y-4 \quad 2x + 3(y-4) = 5$$

$$(7) \quad (2, -8) \rightarrow (-1, -12)$$

$$(8) \quad x = \frac{y}{3y-1} \quad y(3x-1) = x$$

$$x(3y-1) = y \quad y = \frac{x}{3x-1}$$

$$3xy - x = y \quad f^{-1}(x) = \frac{x}{3x-1}$$

$$3xy - y = x$$

$$(9) \quad y = \frac{2x}{1-x} \quad x = xy + 2y$$

$$x = \frac{2y}{1-y} \quad x = y(x+2)$$

$$x - xy = 2y \quad y = \frac{x}{x+2}$$

$$f^{-1}(x) = \frac{x}{x+2}$$

$$(10) \quad y = f(2-x) = f[-(x-2)]$$

$$(-3, 0) \rightarrow (3, 0) \rightarrow (5, 0)$$

$$(0, 0) \rightarrow (0, 0) \rightarrow (2, 0)$$

$$(2, 0) \rightarrow (-2, 0) \rightarrow (0, 0)$$

$$(11) \quad y = \frac{1}{2} f(-(x+1)) - 2$$

$$(3, -4) \rightarrow (3, -2) \rightarrow (-3, -2) \rightarrow (-2, -2) \rightarrow (-2, -4)$$

$$(12) \quad (m, n) \rightarrow (m, -n) \rightarrow \left(\frac{m}{4}, -n\right) \rightarrow \left(\frac{m}{4} + \frac{1}{2}, -n\right) \rightarrow \left(\frac{2+m}{4}, -n+1\right)$$

$$(13) \quad x \rightarrow \frac{1}{3}x \quad \left(\frac{x}{3}\right)^2 + (2y)^2 = 1$$

$$y \rightarrow 2y$$

Ch. 3 Radical functions and Equations

① a) $\sqrt{x-2} = 2$; $x \geq 2$

$$x-2 = 4$$

$$x = 6$$

Check: $\sqrt{6-2} = 2$
 $2 = 2 \checkmark$

$$S = \{6\}$$

b) $\sqrt{5x-6} = x$

$$5x-6 = x^2$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2 \text{ or } x = 3$$

Check: $\sqrt{5 \cdot 2 - 6} = 2$
 $2 = 2 \checkmark$

$$\sqrt{5 \cdot 3 - 6} = 3$$

$$3 = 3 \checkmark$$

$$S = \{2, 3\}$$

c) $\sqrt{13-x} = x-1$

$$13-x = x^2 - 2x + 1$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4 \text{ or } x = -3$$

Check: $\sqrt{13-4} = 4-1$
 $3 = 3 \checkmark$

$$\sqrt{13+3} = -3-1 \quad \times$$

$$S = \{4\}$$

d) $\sqrt[3]{x+6} =$

$$x+6 = 8$$

$$x = 2$$

Check \checkmark

$$S = \{2\}$$

② $y = -2\sqrt{4-2x} + 3$

$$4-2x \geq 0$$

$$4 \geq 2x$$

$$x \leq 2$$

$$d = \{x \mid x \leq 2, x \in \mathbb{R}\}$$

$$r = \{y \mid y \leq 3, y \in \mathbb{R}\}$$

$$x=0 \quad y = -2 \cdot 2 + 3 = -1 \quad (0, -1)$$

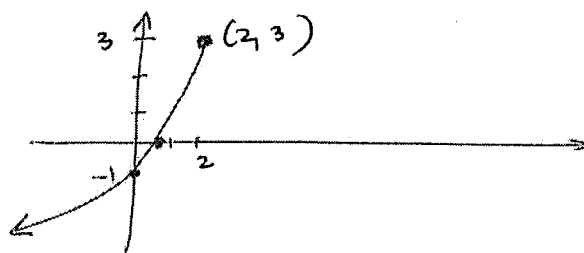
$$y=0 \quad -3 = -2\sqrt{4-2x}$$

$$\left(\frac{3}{2}\right)^2 = 4-2x$$

$$4 - \frac{9}{4} = 2x \quad \left(\frac{7}{8}, 0\right)$$

$$2x = \frac{7}{4}$$

$$x = \frac{7}{8}$$



③ $y = -2\sqrt{x^2-4} + 1$

$$x^2-4 \geq 0 \Rightarrow x \leq -2 \text{ or } x \geq 2$$

$$d = \{x \mid x \leq -2 \text{ or } x \geq 2, x \in \mathbb{R}\}$$

$$r = \{y \mid y \leq 1\}$$

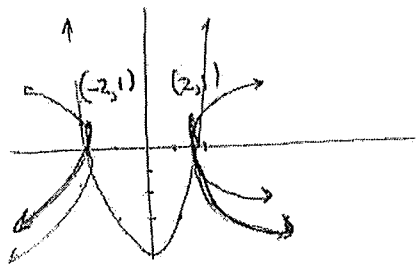
$x=0$ no y-intercept

$$y=0 \quad -1 = -2\sqrt{x^2-4}$$

$$\frac{1}{4} = x^2-4$$

$$x^2 = \frac{17}{4}$$

$$x = \pm \frac{\sqrt{17}}{2}$$



Ch. 2 Polynomial Functions

(1) 5

(2) e

(3) a

(4) e

(5) $y = -\frac{1}{2}(x-6)(x-2)(x+3)$

(6) $-4, 0, \frac{7}{4}$

(7) -135

(8) 3

(9) 5

(10) 72

(11) 24

(12) 0

(13) $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}$

(14) 8


(15) $-3, \frac{2}{5}, \frac{3}{4}$

(16) $-\frac{1}{2}, \frac{-3 \pm \sqrt{7}}{2}$

(17) -2

(18) $a < -4$ or $a > 2$

(19) $x < 0$ or $x > 3$

(20) 

(21) $-4 < x < 2$ or $x > 6$

(22) $x \leq -3$ or $2 \leq x \leq 5$

CH. 4

① $3^{3x} \cdot 3^{2(2x-1)} = 3^{x+4}$
 $3x + 4x - 2 = x + 4$
 $6x = 6$
 $x = 1$

② $A = A_0 x^{\frac{t}{T}}$
 a) $A = 40 \left(\frac{1}{2}\right)^{\frac{48}{14.9}} = 4.3g$
 b) $2.5 = 40 \left(\frac{1}{2}\right)^{\frac{t}{14.9}}$
 $t = 59.6h$

③ a) $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T}}$
 b) 64 times
 c) 3.5 days
 d) 7 days ago
 e) 252 days

④ 6 ⑤ $40 = 100 \left(0.97\right)^{\frac{t}{5}}$ ⑥ $x = 9^{\frac{1}{2}} = 3$

⑦ $(x+1)\log 5 = \log 2 + 2x \log 3$
 $x = \frac{\log 2 - \log 5}{\log 5 - 2\log 3}$

⑧ $\log_a b = 3$ ⑨ $-2x + 6 > 0$
 $x < 3$

⑩ $\frac{\log_4 30}{\log_4 5}$

⑪ $\log x - \log 2 - 3\log y$

⑫ $3^{-2x} = 3^{3(2-x)}$
 $-2x = 6 - 3x$
 $x = 6$

⑬ $x(x-1) = 8$ $x > 0$
 $x^2 - x - 8 = 0$
 $x = \frac{1 \pm \sqrt{33}}{2}$
 $x = \frac{1 + \sqrt{33}}{2}$

⑭ $y = 3^{x-h} - 4$
 $x=0 \quad 5 = 3^{-h} - 4$
 $9 = 3^{-h}$
 $h = -2$
 $y = 3^{x+2} - 4$

⑮ $A = 12000 \left(\frac{1}{3}\right)^{\frac{t}{8}}$
 $\frac{300000}{12000} = \frac{12000}{12000} \cdot 3^{\frac{t}{8}}$
 $t =$

⑯ $\log \frac{m}{nk^3}$

⑰ $\begin{cases} 5-x > 0 \\ x-1 > 0 \\ x \neq 1 \end{cases} \rightarrow 1 < x < 5, x \neq 2$

⑱ a

⑲ $A = 100 \left(5\right)^{\frac{k}{30}}$

⑳ $x = 3 \cdot 2^{\frac{y-2}{5} + 5}$
 $\frac{x-5}{3} = 2^{\frac{y-2}{5}}$
 $y-2 = \log_2 \left(\frac{x-5}{3}\right)$

$y = f^{-1}(x) = \log_2 \left(\frac{x-5}{3}\right) + 2$

㉑ $\frac{(x+4)^2}{-x} = 9$ $\begin{cases} x+4 > 0 \\ -x > 0 \end{cases}$
 $x^2 + 8x + 16 + 9x = 0$
 $x^2 + 17x + 16 = 0$
 $(x+16)(x+1) = 0$
 $x = -16, x = -1$
 reject

㉒ $(2a)^t = p$

㉓ $\log \frac{a}{b^2 c^3}$

㉔ $\frac{3x}{x-3} = 25$
 $3x = 25x - 75$
 $x = \frac{75}{22}$

㉕ $2x+4 = 4x-3+5$
 $x = \frac{2}{2} = 1$

$$(26) \quad 2.6$$

$$(27) \quad 28.20 \text{ days}$$

$$(28) \quad b^x = \frac{c}{a}$$
$$x = \log_b \frac{c}{a}$$

$$(29) \quad x \log 2 = \log 3 + (x+1) \log 5$$

$$x = \frac{\log 3 + \log 5}{\log 2 - \log 5}$$

$$(30) \quad 9 - x^2 = 5 \quad \begin{cases} 3-x > 0 \\ 3+x > 0 \end{cases}$$

$$x^2 = 4$$
$$\boxed{x = 2 \text{ or } -2}$$

$$(31) \quad 3 + (-1) = \log_{\sqrt{4}} x$$

$$2 = \log_{\sqrt{4}} x$$

$$x = 4^2 = 16$$

$$(32) \quad \log_4 (\log_5 x) = \frac{1}{2}$$

$$\log_5 x = 4^{\frac{1}{2}} = \sqrt{4} = 2$$

$$x = 25$$

$$(33) \quad x = 6$$

$$(34) \quad \log_2 \frac{8\sqrt{x}}{y^3}$$

$$(35) \quad \log_{\sqrt{16}} x = \frac{\log_4 x}{\log_4 16} = \frac{a}{2}$$

$$(36) \quad \log 2400 = \log (8 \cdot 3 \cdot 100)$$

$$= \log 8 + \log 3 + \log 100$$

$$= 3 \log 2 + \log 3 + 2$$

$$= 3a + b + 2$$

$$(37) \quad a^{\log_a 16} = 16$$

$$(38)$$

$$\log_n a + 2 \log_n b =$$

$$= 5 + 2 \cdot 3 = 11$$

$$(1) (3 \cos x - 2)(\cos x - 2) = 0$$

$$\cos x = \frac{2}{3} \quad \cos x = 2 \quad \text{no solutions}$$

$$x_1 = \cos^{-1}\left(\frac{2}{3}\right) = 0.8411$$

$$x_2 = 2\pi - \cos^{-1}\left(\frac{2}{3}\right) = 5.4421$$

$$0.8411 + 2n\pi, \quad 5.4421 + 2n\pi, \quad n\text{-integer}$$

$$(3) l = R\theta_{\text{rad}}$$

$$l = 20 \cdot \frac{3\pi}{4} = 15\pi \text{ cm}$$

$$(4) a) \sec \frac{4\pi}{3} = -\frac{2}{\sqrt{3}}$$

$$b) \tan \frac{7\pi}{6} = +\frac{\sqrt{3}}{3}$$

$$c) \sin\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$(5) a) \sin x = \frac{1}{2}$$

$$x_1 = \frac{\pi}{6}$$

$$x_2 = \frac{5\pi}{6}$$

$$b) \sin 2x = \frac{1}{\sqrt{2}}$$

$$\sin a = \frac{1}{\sqrt{2}}$$

$$a_1 = \frac{\pi}{4}$$

$$a_2 = \frac{3\pi}{4}$$

$$x_1 = \frac{\pi}{8}, \quad x_2 = \frac{3\pi}{8}$$

$$x_3 = \frac{9\pi}{8}, \quad x_4 = \frac{11\pi}{8}$$

$$(6) \sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \quad x = \frac{3\pi}{2}$$

$$(7) \tan x (2\cos x - \sqrt{3}) = 0$$

$$\tan x = 0 \quad \cos x = \frac{\sqrt{3}}{2}$$

$$x_1 = 0 \quad x_2 = \frac{\pi}{6} \quad x_3 = -\frac{\pi}{6}$$

$$(8) \pi, \pi + 6n\pi, 2\pi + 6n\pi, n \in \mathbb{Z}$$

$$(9) 0.26 + \frac{2n\pi}{3}, \quad 0.79 + \frac{2n\pi}{3}, \quad n \in \mathbb{Z}$$

$$(10) \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi, n \in \mathbb{Z}$$

$$(11) \frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi, \frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi, n$$

$$\quad \quad \quad \checkmark$$

$$\quad \quad \quad \frac{\pi}{2} + n\pi$$

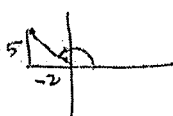
$$(12) \begin{cases} \cos \theta \neq \frac{1}{2} \\ \cos \theta \neq 0 \end{cases}$$

$$(13) \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\sin \theta} + \frac{1}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin \theta} = \frac{1}{\cos^2 \theta \sin \theta} = \sec^2 \theta \cdot \csc \theta$$

$$(14) 3 \cos(x)$$

$$(15) \frac{6 \sin \theta}{2 \sin \theta \cos \theta} = \frac{3}{\cos \theta} = 3 \sec \theta$$

$$(17) \sec \theta = -\frac{\sqrt{29}}{2}$$


$$(18) a = 3 \quad y = -3 \cos 2\left(x - \frac{\pi}{6}\right) + 2$$

$$P = \frac{2\pi}{2} = \pi$$

$$\text{Max} = 2 + 3 = 5$$

$$\text{Min} = 2 - 3 = -1$$

(7) COMBINATORICS

$$(1) 30 C_5$$

$$(2) T_4 = {}_6 C_3 (3x)^3 (-2)^3$$

$$= 20 \cdot 27x^3 \cdot (-8)$$

$$= -4320x^3$$

$$(3) 3360$$

$$(4) 181440$$

$$(5) 3744$$

$$(6) \frac{10!}{7!} = 720$$

$$(7) -80x^2y^3$$

$$(8) n = 7$$

$$(9) \frac{33!}{5!28!}$$

$$(10) T_6 = {}_{10} C_5 (x)^5 (-y)^5$$

$$= -252x^5y^5$$

$$(11) \text{ a) } 4060$$

$$\text{ b) } 24360$$

$$\text{ c) } 1900$$

$$(12) 8$$

$$(13) 210$$

$$(14) 2496$$

$$(15) 60$$

$$(16) 126$$

$$(17) 45$$

$$(18) 201058$$

$$(19) 1728$$