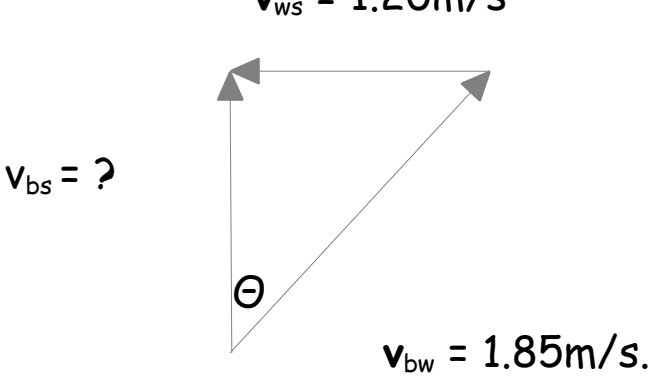
Physics 12 Section 3-8
Relative Velocity

1. Vectors can be used to solve two classic types of motion problems -river crossing and airplane with crosswind.

Example of the river crossing problem:

A boat's speed in still water is $\mathbf{v}_{bw}$ = 1.85m/s. If the boat is to travel directly across a river whose current has a speed $\mathbf{v}_{ws}$ = 1.20m/s, at what upstream angle must the boat head?

1. Draw a picture of what is happening. Label the vectors with subscripts. $\mathbf{v}_{ws}$ is the velocity of the water with respect to (wrt) the shore (current). $\mathbf{v}_{bw}$ is the velocity of the boat wrt the water. v_{bs} is the velocity of the boat wrt the shore.



2. Make a vector statement to show the order of vector addition.

$$\mathbf{v}_{bw} + \mathbf{v}_{ws} = \mathbf{v}_{bs}$$

3. Solve for the resulting vector (magnitude and direction) using trigonometry or Pythagoras.

$$\sin \Theta = \frac{1.20}{1.85}$$

$$\Theta = \sin^{-1} \frac{1.20}{1.85}$$

$$\Theta = 40.4^\circ$$

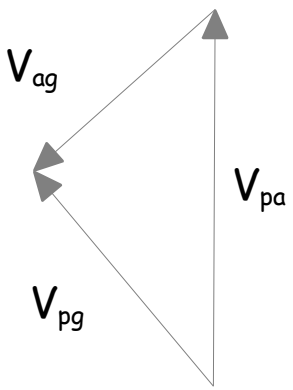
$$\tan \Theta = \frac{1.20}{v_{bs}}$$

$$v_{bs} = \frac{1.20}{\tan 40.4^\circ}$$

$$v_{bs} = 1.41 \text{ m/s}$$

1.41 m/s directly across the river.

2. An airplane whose airspeed is 200km/h heads due North. A 100km/h North East wind suddenly begins to blow. What is the resulting velocity of the plane with respect to the ground?



V_{ag} = velocity of the wind wrt ground.
V_{pa} = velocity of the plane wrt the air.
V_{pg} = velocity of the plane wrt the ground.

$$V_{pa} + V_{ag} = V_{pg}$$

cosine law will solve for the magnitude and the sine law will solve for the angle.