Maximum Range Formula Derivation

Set $x_o = 0$
$y_o = 0$
$y = 0$

$$y = y_o + v_{yo}t - \tfrac{1}{2} gt^2$$

$$0 = v_{yo}t - \tfrac{1}{2} gt^2$$

Solving for t we get $t = 0$ or $t = 2v_{yo}/g$

Range $= x = v_{xo}t$

$$R = (v_{xo})(2v_{yo})/g$$

$$R = 2v_{xo}v_{yo}/g$$

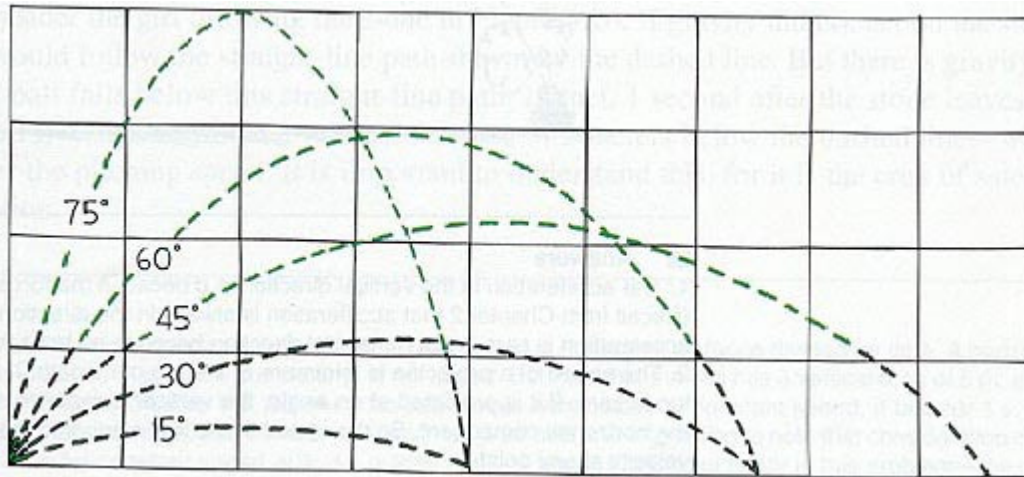$$v_{xo} = v_o \cos \theta$$

$$v_{yo} = v_o \sin \theta$$

Substitute

$$R = \frac{2v_o^2 (\sin \theta)(\cos \theta)}{g}$$

$2\sin \theta \cos \theta = \sin2\theta$ from trig formula

$$R = \frac{v_o^2 \sin2\theta}{g}$$

Range versus Firing Angle

Two angles whose sum is 90 degrees each give the same range.

At what angle is maximum range achieved?