

40. (a) Using the coordinate system shown, for momentum conservation we have

$$\begin{aligned} x\text{-momentum: } mv + 0 &= 0 + 2mv_1' \cos \theta, \text{ or} \\ 2v_1' \cos \theta &= v \end{aligned}$$

$$\begin{aligned} y\text{-momentum: } 0 + 0 &= -mv_1' + 2mv_1' \sin \theta, \text{ or} \\ 2v_1' \sin \theta &= v_1' \end{aligned}$$

If we square and add these two equations, we get

$$v^2 + v_1'^2 = 4v_1'^2$$

For the conservation of kinetic energy, we have

$$\begin{aligned} \frac{1}{2}mv^2 + 0 &= \frac{1}{2}mv_1'^2 + \frac{1}{2}(2m)v_1'^2, \text{ or} \\ v^2 - v_1'^2 &= 2v_1'^2 \end{aligned}$$

When we add this to the previous result, we get

$$v^2 = 3v_1'^2$$

Using this in the x -momentum equation, we get

$$\cos \theta = \sqrt{3}/2, \text{ or } \theta = \boxed{30^\circ}$$

- (b) From part (a) we have

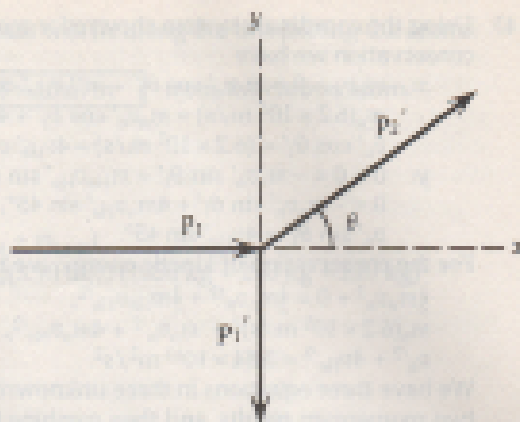
$$v_1' = v/\sqrt{3}$$

Using the energy result, we get

$$v_1'^2 = v^2 - 2v_1'^2 = v^2 - 2v^2/3 = \frac{1}{3}v^2, \text{ or } \boxed{v_1' = v/\sqrt{3}}$$

- (c) The fraction of the kinetic energy transferred is

$$\text{fraction} = K_2/K_1 = \frac{1}{2}(2m)v_1'^2 / \frac{1}{2}mv^2 = mv^2/3 / \frac{1}{2}mv^2 = \boxed{\frac{2}{3}}$$



41. Using the coordinate system shown, for momentum conservation we have

$$y\text{-momentum: } -mv \sin \theta_1 + mv \sin \theta_2 = 0, \text{ or } \theta_1 = \theta_2$$

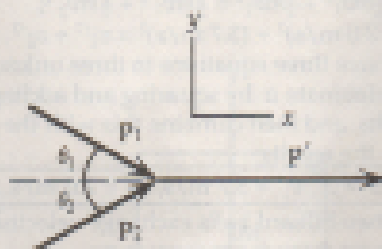
$$x\text{-momentum: } mv \cos \theta_1 + mv \cos \theta_2 = 2mv_1'$$

$$2mv \cos \theta_1 = 2mv_1'/3$$

$$\cos \theta_1 = \frac{1}{3}, \text{ or } \theta_1 = 70.5^\circ = \theta_2$$

The angle between their initial directions is

$$\theta = \theta_1 + \theta_2 = 2(70.5^\circ) = \boxed{141^\circ}$$



42. Using the coordinate system shown, for momentum conservation we have

$$y\text{-momentum: } mv_1 + 0 = mv_1' \cos \alpha + Mv_2' \cos \theta_2$$

$$5M(12.0 \text{ m/s}) = 5Mv_1' \cos \alpha + Mv_2' \cos 80^\circ, \text{ or}$$

$$5v_1' \cos \alpha = -v_2' \cos 80^\circ + 60.0 \text{ m/s}$$

$$x\text{-momentum: } 0 = -mv_1' \sin \alpha + Mv_2' \sin \theta_2$$

$$0 = -5Mv_1' \sin \alpha + Mv_2' \sin 80^\circ, \text{ or}$$

$$5v_1' \sin \alpha = v_2' \sin 80^\circ$$

For the conservation of kinetic energy, we have

$$\frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_1'^2 + \frac{1}{2}Mv_2'^2$$

$$5M(12.0 \text{ m/s})^2 = 5Mv_1'^2 + Mv_2'^2, \text{ or}$$

$$5v_1'^2 + v_2'^2 = 720 \text{ m}^2/\text{s}^2$$

We have three equations in three unknowns: α , v_1' , v_2' . We eliminate α by squaring and adding the two momentum results, and then combine this with the energy equation, with the results:

(a) $v_2' = \boxed{3.47 \text{ m/s}}$

(b) $v_1' = \boxed{11.9 \text{ m/s}}$

(c) $\alpha = \boxed{3.29^\circ}$

