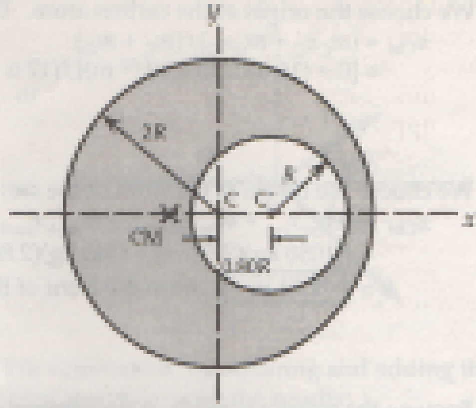
51. We know from the symmetry that the center of mass lies on a line containing the center of the plate and the center of the hole. We choose the center of the plate as origin and x along the line joining the centers. Then $y_{CM} = 0$. A uniform circle has its center of mass at its center. We can treat the system as two circles:



a circle of radius $2R$, density ρ and mass $\rho\pi(2R)^2$ with $x_1 = 0$;

a circle of radius R, density $-\rho$ and mass $-\rho\pi R^2$ with $x_2 = 0.80R$.

We find the center of mass from

$$x_{CM} = (m_1x_1 + m_2x_2)/(m_1 + m_2)$$
$$= [4\rho\pi R^2\,(0) - \rho\pi R^2\,(0.80R)]/(4\rho\pi R^2 - \rho\pi R^2)$$
$$= -0.27R.$$

The center of mass is **along the line joining the centers 0.07R outside the hole.**

52. If we assume a total mass of 70 kg, for one leg we have

$$m_{leg} = m_{body}\tfrac{1}{2}(21.5 + 9.6 + 3.4)/100 = (70\text{ kg})\tfrac{1}{2}(34.5)/100 = \boxed{12\text{ kg.}}$$

53. If we measure from the shoulder, the percentage of the height to the center of mass for each of the segments is

upper arm: 81.2 – 71.7 = 9.5;

lower arm: 81.2 – 55.3 = 25.9;

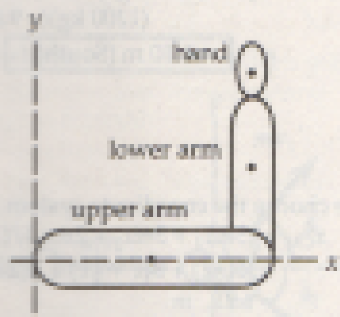
hand: 81.2 – 43.1 = 38.1.

Because all masses are percentages of the body mass, we can use the percentages rather than the actual mass. Thus we have

$$x_{CM} = (m_{upper}x_{upper} + m_{lower}x_{lower} + m_{hand}x_{hand})/(m_{upper} + m_{lower} + m_{hand})$$
$$= [\tfrac{1}{2}(6.6)(9.5) + \tfrac{1}{2}(4.2)(25.9) + \tfrac{1}{2}(1.7)(38.1)]/[\tfrac{1}{2}(6.6) + \tfrac{1}{2}(4.2) + \tfrac{1}{2}(1.7)]$$
$$= 19.$$

The CM of an outstretched arm is **19% of the height.**

54. We choose the shoulder as the origin. The locations of the centers of mass for each of the segments are



upper arm: $x_{upper} = [(81.2 - 71.7)/100](155\text{ cm}) = 14.7\text{ cm}$;

$y_{upper} = 0$;

lower arm: $x_{lower} = [(81.2 - 62.2)/100](155\text{ cm}) = 29.5\text{ cm}$;

$y_{lower} = [(62.2 - 55.3)/100](155\text{ cm}) = 10.7\text{ cm}$;

hand: $x_{hand} = [(81.2 - 62.2)/100](155\text{ cm}) = 29.5\text{ cm}$;

$y_{hand} = [(62.2 - 43.1)/100](155\text{ cm}) = 29.6\text{ cm}$.

Because all masses are percentages of the body mass, we can use the percentages rather than the actual mass. Thus we have

$$x_{CM} = (m_{upper}x_{upper} + m_{lower}x_{lower} + m_{hand}x_{hand})/(m_{upper} + m_{lower} + m_{hand})$$
$$= [\tfrac{1}{2}(6.6)(14.7\text{ cm}) + \tfrac{1}{2}(4.2)(29.5\text{ cm}) + \tfrac{1}{2}(1.7)(29.5\text{ cm})]/[\tfrac{1}{2}(6.6) + \tfrac{1}{2}(4.2) + \tfrac{1}{2}(1.7)]$$
$$= \boxed{22\text{ cm.}}$$
$$y_{CM} = (m_{upper}y_{upper} + m_{lower}y_{lower} + m_{hand}y_{hand})/(m_{upper} + m_{lower} + m_{hand})$$
$$= [\tfrac{1}{2}(6.6)(0) + \tfrac{1}{2}(4.2)(10.7\text{ cm}) + \tfrac{1}{2}(1.7)(29.6\text{ cm})]/[\tfrac{1}{2}(6.6) + \tfrac{1}{2}(4.2) + \tfrac{1}{2}(1.7)]$$
$$= \boxed{7.6\text{ cm.}}$$